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Article

Dirac Field of Negative Energy and Primordial Antimatter Incursion

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Abstract: A Dirac field of negative energy is introduced. It is shown how such a negative energy field is connected to the Dirac field of positive energy. We also show that electric charges of the negative energy field are exactly opposite to those of the positive energy field. Finally we show, within the framework of QFT, how a fermion with a positive energy and mass state can be transformed into a fermion whose energy and mass state is negative – and vice versa – through the use of an electromagnetic potential. This gives another possible explanation for the excess of antiprotons observed in the AMS-02 experiment in 2016, as a primordial antimatter incursion.

Keywords: Negative mass, negative energy, Dirac field, primordial antimatter, quantum field theory, mass inversion, dark matter

1. Introduction

As an answer to CP-violation, C asymmetry and the baryon asymmetry of the universe (i.e. the missing primordial antimatter), Sakharov in 1967 [1] introduced the idea of two universes with opposite arrows of time, born from the same initial singularity (i.e. the Big Bang). In this way, CPT symmetry is preserved as a whole. In 2001, Petit et al. [2] have established that time reversal in one of Sakharov's twin universes identifies to energy (and mass) inversion, using Souriau's prior work (1970) in dynamical group theory [3]. In 2018, Debergh et al. [4] have shown that the exclusion of negative energy states from quantum mechanics, through the use of an anti-unitary and anti-linear T (time reversal) operator, is an arbitrary axiom and that it is perfectly allowed to use a unitary and linear T operator instead. They argue that there are two kinds of antimatter: the "Dirac antimatter", created in the lab, is associated with an anti-unitary PT transformation; while the missing one or "Feynman antimatter" belongs to Sakharov's twin universe and is associated with a unitary PT transformation. The former has positive energy and mass, while the latter has negative ones. Therefore, negative mass populating Sakharov's twin universe may be a substitute for dark matter, as considered in the Janus cosmological model (JCM) [5, 6, 7]. For Sakharov, the two universes are related only by their common initial singularity. In JCM, it is also assumed that the two universes interact everywhere and at all times, gravitationally (more exactly, anti-gravitationally). JCM generalizes Einstein's theory of gravitation to include negative mass in cosmology using two metrics instead of one, eliminating the runaway motion [8], a preposterous effect often used to reject the existence of negative energy, even if Dirac's theory does not prohibit such state [9, 10]. As negative energy photons follow null geodesics of their own metric and do not interact with positive energy ones, negative mass appears optically invisible, or dark. Although other bimetric theories of gravity also exist, such as [11, 12, 13, 14, 15], the JCM is in good agreement with many observational data [5, 6] and gives the first coherent description of "the dipole repeller" [16].

Within the framework of Dirac's QFT, taking into account a unitary time reversal transformation, we introduce a Dirac field of negative energy and show that a fermion can be transferred from one universe to the other. Actually, this is a physical mass inversion process. Quantum states of positive and negative energies (i.e. masses $+\mu$ and $-\mu$) can be coupled by using an electromagnetic potential. The latter allows a transition from one state to another to occur with a non-zero probability. As our goal is to show the practicability of mass inversion, we need to avoid using perturbative computation because mass inversion implies a quantum resonance effect which manifests itself only if the potential is relatively large compared to the energy of the relativistic particle. This is why we limit ourselves here to a uniform and time-independent electromagnetic potential. In fact, such a potential does act on a charged particle, as shown by the Aharonov-Bohm effect [17, 18] in regions of space where the electromagnetic field is zero.

The physical mass inversion presented herein is a foundation stone that could give other researchers the opportunity to eventually popper-falsify the Janus model. Indeed, according to JCM, if a mass $+\mu$ becomes $-\mu$, the particle will "fall up", repelled by Earth's gravitational field. Also, such mass inversion would offer an alternate explanation for dark matter annihilation, measured as an excess of antiprotons observed in the AMS-02 experiment in 2016 [19]. Instead, this measured excess could be due to the incursion in our universe of some primordial antimatter (i.e. antiprotons) populating Sakharov's "twin universe", after its mass inversion from $-\mu$ to $+\mu$.

2. Dirac Field in Flat Spacetime

The Dirac field equation for free particles with a positive or negative mass in covariant form is [20]:

$$\gamma^{\nu}\frac{\partial}{\partial x^{\nu}}\Psi_{\pm}\pm i\kappa\Psi_{\pm}=0 \tag{1}$$

where $\kappa = c\mu/\hbar$. c and μ are the speed of the light in vacuum and the rest mass, respectively. Dirac fields operators with mass $+\mu$ and $-\mu$ are given respectively by:

$$\Psi_{+}(\mathbf{r})=\int d^{3}\mathbf{p}\sum_{S}\left(\Phi_{+,S,\mathbf{p}}(\mathbf{r})\mathrm{a}_{+,S,\mathbf{p}}+\Phi_{-,S,\mathbf{p}}(\mathbf{r})\,\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}\right), \tag{2}$$

and

$$\Psi_{-}(\mathbf{r})=\int d^{3}\mathbf{p}\sum_{S}\left(\Theta_{-,S,\mathbf{p}}(\mathbf{r})\mathrm{b}_{-,S,\mathbf{p}}+\Theta_{+,S,\mathbf{p}}(\mathbf{r})\,\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}\right). \tag{3}$$

$\mathrm{a}_{j,S,\mathrm{p}}(\mathrm{b}_{j,S,\mathrm{p}})$ and $\mathrm{a}^{\dagger}_{j,S,\mathrm{p}}$ $(\mathrm{b}^{\dagger}_{j,S,\mathrm{p}})$ are the operators of annihilation and creation of fermions of positive (negative) mass in the occupation number space of the single-particle state, characterized by quantum numbers j, S and $\mathbf{p}$. They obey the usual anti-commutation relations:

$$\{\mathrm{a}_{j,S,\mathbf{p}}\,,\,\mathrm{a}^{\dagger}_{j',S',\mathbf{p}'}\}=\delta_{j,j'}\,\delta_{S,S'}\delta(\mathbf{p}-\mathbf{p}')=\{\mathrm{b}_{j,S,\mathbf{p}}\,,\,\mathrm{b}^{\dagger}_{j',S',\mathbf{p}'}\} \tag{4}$$

$$\{\mathrm{a}^{\dagger}_{j,S,\mathbf{p}}\,,\,\mathrm{a}^{\dagger}_{j',S',\mathbf{p}'}\}=0=\{\mathrm{b}^{\dagger}_{j,S,\mathbf{p}}\,,\,\mathrm{b}^{\dagger}_{j',S',\mathbf{p}'}\}, \tag{5}$$

$$\{\mathrm{a}_{j,S,\mathbf{p}}\,,\,\mathrm{a}_{j',S',\mathbf{p}'}\}=0=\{\mathrm{b}_{j,S,\mathbf{p}}\,,\,\mathrm{b}_{j',S',\mathbf{p}'}\}. \tag{6}$$

The index $j = (+, -)$ characterizes positive and negative frequencies. $S = (R, L)$ is the helicity state. The vector $\mathbf{p}$ is the classical canonical variable of momentum conjugated to the position vector $\mathbf{r}$. $\Phi_{j,S,\mathrm{p}}(\mathrm{r})$ and $\Theta_{j,S,\mathrm{p}}(\mathrm{r})$ are the stationary states of:

$$H_{0}^{(+)}\ \Phi_{j,S,\mathbf{p}}(\mathbf{r})=j\,\varepsilon(\mathbf{p})\ \Phi_{j,S,\mathbf{p}}(\mathbf{r}) \tag{7}$$

and:

$$H_{0}^{(-)}\ \Theta_{j,S,\mathbf{p}}(\mathbf{r})=j\,\varepsilon(\mathbf{p})\ \Theta_{j,S,\mathbf{p}}(\mathbf{r}) \tag{8}$$

with the eigenvalue of energy $\varepsilon(\mathbf{p})$ given by:

$$\varepsilon(\mathbf{p}) = (c^2\mathbf{p}^2 + \mu^2 c^4)^{1/2}\,. \tag{9}$$

The one-particle Dirac Hamiltonian operator $H_o^{(\pm)}$ for a free particle of spin ½ and mass $\pm\mu$ is given by:

$$H_o^{(\pm)} = c\alpha\cdot\mathbf{P} \pm \beta\mu c^2\,. \tag{10}$$

β and α_u(u = x, y, z) are the Dirac matrices:

$$\alpha_u = \begin{pmatrix} 0 & \sigma_u \\ \sigma_u & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix} \tag{11}$$

where σ_u are the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{12}$$

I is the matrix unit 2×2 and $\mathbf{P} \to -i\hbar\nabla$. Orthonormal wave functions can be expressed as follows:

$$\Phi_{j,S,\mathbf{p}}(\mathbf{r}) = \phi_{j,S,\mathbf{p}}\frac{e^{i(\mathbf{p}\cdot\mathbf{r}/\hbar)}}{\sqrt{(2\pi\hbar)^3}} \qquad \Theta_{j,S,\mathbf{p}}(\mathbf{r}) = \theta_{j,S,\mathbf{p}}\frac{e^{i(\mathbf{p}\cdot\mathbf{r}/\hbar)}}{\sqrt{(2\pi\hbar)^3}} \tag{13}$$

where $\phi_{j,S,\mathrm{p}}$ are the two-spinors [20]:

$$\phi_{+,R,\mathbf{p}} = \mathscr{B}(\mathbf{p})\begin{pmatrix} (1+n_z) \\ (n_x + in_y) \\ \frac{cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{cp(n_x+in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \end{pmatrix}, \quad \phi_{+,L,\mathbf{p}} = \mathscr{B}(\mathbf{p})\begin{pmatrix} -(n_x - in_y) \\ (1+n_z) \\ \frac{cp(n_x-in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{-cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \end{pmatrix} \tag{14}$$

$$\phi_{-,R,\mathbf{p}} = \mathscr{B}(\mathbf{p})\begin{pmatrix} \frac{-cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{-cp(n_x+in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ (1+n_z) \\ (n_x + in_y) \end{pmatrix}, \quad \phi_{-,L,\mathbf{p}} = \mathscr{B}(\mathbf{p})\begin{pmatrix} \frac{-cp(n_x-in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ -(n_x - in_y) \\ (1+n_z) \end{pmatrix} \tag{15}$$

and

$$\theta_{+,R,\mathbf{p}} = \mathscr{B}(\mathbf{p}) \begin{pmatrix} \frac{cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{cp(n_x+in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ (1+n_z) \\ (n_x+in_y) \end{pmatrix}, \quad \theta_{+,L,\mathbf{p}} = \mathscr{B}(\mathbf{p}) \begin{pmatrix} \frac{cp(n_x-in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{-cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ -(n_x-in_y) \\ (1+n_z) \end{pmatrix} \tag{16}$$

$$\theta_{-,R,\mathbf{p}} = \mathscr{B}(\mathbf{p}) \begin{pmatrix} (1+n_z) \\ (n_x+in_y) \\ \frac{-cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{-cp(n_x+in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \end{pmatrix}, \quad \theta_{-,L,\mathbf{p}} = \mathscr{B}(\mathbf{p}) \begin{pmatrix} -(n_x-in_y) \\ (1+n_z) \\ \frac{-cp(n_x-in_y)}{[\varepsilon(\mathbf{p})+\mu c^2]} \\ \frac{cp(1+n_z)}{[\varepsilon(\mathbf{p})+\mu c^2]} \end{pmatrix}. \tag{17}$$

(n_x, n_y, n_z) are the three components of the unit vector $\mathbf{p}/p$ with $p\equiv|\mathbf{p}|$. The normalization constant $\mathscr{B}(\mathbf{p})$ is obtained by taking $\phi^{\dagger}_{j,S,p}\phi_{j,S,p} = 1 = \theta^{\dagger}_{j,S,p}\theta_{j,S,p}$:

$$\mathscr{B}(\mathbf{p}) = \frac{1}{2}\sqrt{\frac{\varepsilon(\mathbf{p})+\mu c^2}{\varepsilon(\mathbf{p})(1+n_z)}}\,. \tag{18}$$

Note that $\phi_{j,S,p}$ with $\mu \to -\mu$ are also solutions of $H_o^{(-)}$. But these are trivial solutions: no new quantum state. However, the $\theta_{j,S,p}$ solutions are actually new quantum states. The Hamiltonian of the Dirac field for free particles with positive and negative masses are simply given by [20]:

$$\mathscr{H}_o^{(+)} \equiv \int d^3\mathbf{r}\, \Psi_+^{\dagger}(\mathbf{r}) H_o^{(+)} \Psi_+(\mathbf{r}) + C = \int d^3\mathbf{p} \sum_{j,S} \varepsilon(\mathbf{p})\, a^{\dagger}_{j,S,\mathbf{p}} a_{j,S,\mathbf{p}} \tag{19}$$

$$\mathscr{H}_o^{(-)} \equiv \int d^3\mathbf{r}\, \Psi_-^{\dagger}(\mathbf{r}) H_o^{(-)} \Psi_-(\mathbf{r}) - C = -\int d^3\mathbf{p} \sum_{j,S} \varepsilon(\mathbf{p})\, b^{\dagger}_{j,S,\mathbf{p}} b_{j,S,\mathbf{p}} \tag{20}$$

where

$$C \equiv 2\delta(0) \int d^3\mathbf{p}\, \varepsilon(\mathbf{p})\,. \tag{21}$$

The constant C in (21) is infinite. It is a singularity. But one can see that with negative energy fields, the sum of energies eliminates the constant C. Thus, the singularity vanishes when negative energies are taken into account. Equations (14) and (16) give the two-spinors corresponding to the eigenvalue $j\varepsilon(\mathbf{p}) = \varepsilon(\mathbf{p})$ (positive frequency) while (15) and (17) give those associated with the eigenvalue $j\varepsilon(\mathbf{p}) = -\varepsilon(\mathbf{p})$ (negative frequency). In addition we have:

$$\phi^{\dagger}_{j,S,\mathbf{p}}\phi_{j',S',\mathbf{p}} = \delta_{j,j'}\delta_{S,S'} = \theta^{\dagger}_{j,S,\mathbf{p}}\theta_{j',S',\mathbf{p}} \,. \tag{22}$$

It should be noted that the two-spinors in (14)-(15) and (16)-(17) have a well-defined helicity (*R* and *L*). They are the eigenspinors of the one-particle helicity operator (Σ·**p**)/p with eigenvalues +1 ($S = R$) and −1 ($S = L$):

$$\frac{1}{\mathrm{p}}(\Sigma\cdot\mathbf{p})\phi_{j,S,\mathbf{p}} = \pm\phi_{j,S,\mathbf{p}} \qquad \frac{1}{\mathrm{p}}(\Sigma\cdot\mathbf{p})\theta_{j,S,\mathbf{p}} = \pm\theta_{j,S,\mathbf{p}} \tag{23}$$

where Σ, in standard representation, is given by:

$$\Sigma_u = \begin{pmatrix} \sigma_u & 0 \\ 0 & \sigma_u \end{pmatrix} . \tag{24}$$

Finally we must note that we have two vacua $|\,0\rangle_{\mathrm{a}}$and $|\,0\rangle_{\mathrm{b}}$defined by:

$$\mathrm{a}_{j,S,\mathbf{p}}|0\rangle_{\mathrm{a}} = 0 \qquad \mathrm{b}_{j,S,\mathbf{p}}|0\rangle_{\mathrm{b}} = 0 \,. \tag{25}$$

3. The Meaning of Negative Energy

3.1. Unitary Time Reversal

Negative energy can be see or the result of unitary time-reversal. According to the work of Debergh et al. [4] the two-spinors of negative mass in (16) to (17) can be obtained from the two-spinors of positive mass (14) to (15) when these undergo a unitary time reversal operation such as:

$$\begin{aligned} x &\rightarrow x \\ t &\rightarrow -t \\ \mathrm{i} &\rightarrow \mathrm{i} \\ E &\rightarrow -E \\ \mu &\rightarrow -\mu \\ \mathbf{p} &\rightarrow \mathbf{p} \,. \end{aligned} \tag{26}$$

By changing *t* to −*t* in the energy operator, the energy changes sign since the imaginary number "i" does not change (unitary time reversal):

$$E \leftrightarrow i\hbar\frac{\partial}{\partial t} \rightarrow -i\hbar\frac{\partial}{\partial t} \leftrightarrow -E \ . \tag{27}$$

And since the mass μ is energy at rest ($E = \mu c^2$), unitary time reversal goes with inversion of the mass sign also. Note that such reversal leaves the exponential functions invariant:

$$e^{-i\frac{\varepsilon(\mathbf{p})t}{\hbar}} \rightarrow e^{-i\frac{\varepsilon(\mathbf{p})t}{\hbar}} \quad \text{and} \quad e^{i\frac{\mathbf{p}\cdot\mathbf{r}}{\hbar}} \rightarrow e^{i\frac{\mathbf{p}\cdot\mathbf{r}}{\hbar}} \ . \tag{28}$$

So, by doing $\varepsilon(\mathbf{p}) \rightarrow -\varepsilon(\mathbf{p})$ and $\mu \rightarrow -\mu$ in (14) to (15) we find equations (16) to (17) as:

$$\text{T} - \text{unitary}: \qquad \phi_{\pm,S,\mathbf{p}} \rightarrow \theta_{\mp,S,\mathbf{p}} \ . \tag{29}$$

For comparison, an anti-unitary time reversal operation leads to:

$$\begin{aligned} x &\rightarrow x \\ t &\rightarrow -t \\ \mathrm{i} &\rightarrow -\mathrm{i} \\ E &\rightarrow E \\ \mu &\rightarrow \mu \\ \mathbf{p} &\rightarrow -\mathbf{p} \ . \end{aligned} \tag{30}$$

The last relation is the result of the operator:

$$\mathbf{P} \leftrightarrow -i\hbar\nabla \ . \tag{31}$$

3.2. Opposite Phase Factor

Negative energy can also be seen as the result of taking an opposite phase factor for a quantum state. To see it, let's rewrite the two-spinors $\phi_{j,S,\mathrm{p}}$ and $\theta_{j,S,\mathrm{p}}$. The basis of the representation used above for the two-spinors is the vector product of two bases of states and is of dimension 4: $|\eta\rangle_1 \otimes |\eta'\rangle_2 \equiv |\eta\rangle_1 |\eta'\rangle_2$ and they are given by:

$$|+\rangle_1|+\rangle_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \; |+\rangle_1|-\rangle_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \; |-\rangle_1|+\rangle_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \; |-\rangle_1|-\rangle_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} . \tag{32}$$

These equalities come from the following prescription:

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} a\alpha \\ a\beta \\ b\alpha \\ b\beta \end{pmatrix} \tag{33}$$

which gives a two-spinors from the vector product of two spinors and definitions:

$$|+\rangle_{\rm k} \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{34}$$

$$|-\rangle_{\rm k} \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{35}$$

where k = 1,2. Let's define two pairs of spinor states:

$$\begin{aligned} |+\rangle_{\rm II} &= \cos(\theta/2)\,|+\rangle_2 + \sin(\theta/2)e^{i\phi}\,|-\rangle_2 \\ |-\rangle_{\rm II} &= -\sin(\theta/2)e^{-i\phi}\,|+\rangle_2 + \cos(\theta/2)\,|-\rangle_2 \end{aligned} \tag{36}$$

which is the spin or helicity along momentum **p** and

$$|+\rangle_{\mathrm{I}} = \cos(\varphi/2)\,|+\rangle_{1} + \sin(\varphi/2)e^{i\chi}\,|-\rangle_{1}$$
$$|-\rangle_{\mathrm{I}} = -\sin(\varphi/2)e^{-i\chi}\,|+\rangle_{1} + \cos(\varphi/2)\,|-\rangle_{1}\,. \tag{37}$$

Taking into account the definitions, $\mathbf{p} = (p_x, p_y, p_z)$ and $n_u = p_u/|\mathbf{p}|$, we set:

$$n_x = \sin(\theta)\cos(\phi)$$
$$n_y = \sin(\theta)\sin(\phi)$$
$$n_z = \cos(\theta) \tag{38}$$

where $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. Parameter φ is defined by the relations:

$$\cos(\varphi) = \frac{\mu c^2}{\varepsilon(\mathbf{p})}$$
$$\sin(\varphi) = \frac{c|\mathbf{p}|}{\varepsilon(\mathbf{p})}\,, \tag{39}$$

with $0 \le \varphi \le \pi/2$ and where $0 \le \chi \le 2\pi$. In accordance with the definitions given above, the equations (14)-(15) and (16)-(17) take simple forms:

$$\phi_{+,R,\mathbf{p}} = |+\rangle_{\mathrm{I}_{(\chi=0)}}\,|+\rangle_{\mathrm{II}} \qquad \theta_{-,R,\mathbf{p}} = |+\rangle_{\mathrm{I}_{(\chi=\pi)}}\,|+\rangle_{\mathrm{II}} \tag{40}$$

$$\phi_{+,L,\mathbf{p}} = |+\rangle_{\mathrm{I}_{(\chi=\pi)}}\,|-\rangle_{\mathrm{II}} \qquad \theta_{-,L,\mathbf{p}} = |+\rangle_{\mathrm{I}_{(\chi=0)}}\,|-\rangle_{\mathrm{II}} \tag{41}$$

$$\phi_{-,R,\mathbf{p}} = |-\rangle_{\mathrm{I}_{(\chi-0)}}|+\rangle_{\mathrm{II}} \qquad \theta_{+,R,\mathbf{p}} = |-\rangle_{\mathrm{I}_{(\chi-\pi)}}\,|+\rangle_{\mathrm{II}} \tag{42}$$

$$\phi_{-,L,\mathbf{p}} = |-\rangle_{\mathrm{I}_{(\chi=\pi)}}|-\rangle_{\mathrm{II}} \qquad \theta_{+,L,\mathbf{p}} = |-\rangle_{\mathrm{I}_{(\chi=0)}}\,|-\rangle_{\mathrm{II}}. \tag{43}$$

According to (40)-(43), the phase factor χ of the states$\theta_{-j,S,\mathrm{p}}$has an opposite value (i.e. $\pm\pi$) relatevely to the $\phi_{j,S,\mathrm{p}}$ones. This is equivalent to reversing the sign of the mass and the energy in the $\phi_{j,S,\mathrm{p}}$ or taking into account a unitary time reversal.

4. Connection between Fermions of Mass +*μ* and -*μ*

From (14) to (17) it can be shown that:

$$\begin{pmatrix} \theta_{+,S,\mathbf{p}} \\ \theta_{-,S,\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \eta\sin(\varphi) & \cos(\varphi) \\ \cos(\varphi) & -\eta\sin(\varphi) \end{pmatrix} \begin{pmatrix} \phi_{+,S,\mathbf{p}} \\ \phi_{-,S,\mathbf{p}} \end{pmatrix} \tag{44}$$

with η = +1 for $S = R$ and -1 for $S = L$. So, there is a relation between the two kinds of two-spinors which is quite similar to a rotation in a plane. But the direction of rotation changes with the helicity. Because of these relations, there are similar relationships between operators $a_{j,S,\mathrm{p}}$ and $b_{j,S,\mathrm{p}}$. If we put (44) in equation (3) we can show by identification with (2) that:

$$\begin{pmatrix} a^{\dagger}_{-,S,-\mathbf{p}} \\ a_{+,S,\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \cos(\varphi) & -\eta\sin(\varphi) \\ \eta\sin(\varphi) & \cos(\varphi) \end{pmatrix} \begin{pmatrix} b^{\dagger}_{+,S,-\mathbf{p}} \\ b_{-,S,\mathbf{p}} \end{pmatrix} \tag{45}$$

and then $\Psi_+(\mathrm{r}) = \Psi_-(\mathrm{r}) \equiv \Psi(\mathbf{r})$. Equation (45) is a Bogoliubov transformation. On the other hand, the charge operator Q is defined by [20]:

$$\mathrm{Q} \equiv -\frac{\mathrm{e}}{2}\int \mathrm{d}^3\mathbf{r}\left(\Psi^{\dagger}(\mathbf{r})\Psi(\mathbf{r}) - \widetilde{\Psi}(\mathbf{r})\widetilde{\Psi}^{\dagger}(\mathbf{r})\right) \tag{46}$$

where "$\mathrm{e} > 0$" is the elementary electric charge and symbol "$\sim$" indicates matrix transposition. Therefore, we get from (2)-(3) two representations of Q:

$$\mathrm{Q} = -\mathrm{e}\int \mathrm{d}^3\mathbf{p}\sum_{j,S}\left(j\, a^{\dagger}_{j,S,\mathbf{p}} a_{j,S,\mathbf{p}}\right) = \mathrm{e}\int \mathrm{d}^3\mathbf{p}\sum_{j,S}\left(j\, b^{\dagger}_{j,S,\mathbf{p}} b_{j,S,\mathbf{p}}\right). \tag{47}$$

As we can see in the expression of Q in terms of $a_{j,S,\mathrm{p}}$, we have fermions of negative charge (i.e. j = +) and fermions of positive charge (i.e. j = −). But, in terms of $b_{j,S,\mathrm{p}}$, we have fermions of positive charge (i.e. j = +) and fermions of negative charge (i.e. j = −). The signs of the charges are exactly opposite for a given j. So, index j (i.e. frequency) is also a quantum number for charge.

5. Mass Inversion

The expression "physical mass inversion" is a misuse of language. It does not reverse the mass by acting "directly" on it. We are acting with an electromagnetic potential upon a quantum state where the mass is $+\mu$ so that there is a non-negligible probability that it is changed into another quantum state where the mass is $-\mu$. The opposite is also true.

5.1. Coupling to an Electromagnetic Potential

It is well known [20] that when an electromagnetic potential A_ν imposed from the outside by a macroscopic source acting on a particle with an electric charge q, mass $\pm\mu$ and spin ½, equation (1) becomes:

$$\gamma^{\nu}\left(\frac{\partial}{\partial x^{\nu}}+\frac{iq}{\hbar}A_{\nu}\right)\Psi_{\pm}\pm i\kappa\Psi_{\pm}=0\ . \qquad (48)$$

In order to obtain numerical results it is more convenient to use non-covariant form:

$$i\,\hbar\frac{\partial}{\partial t}\Psi_{\pm}(\mathbf{r},\mathrm{t})=H^{(\pm)}\Psi_{\pm}(\mathbf{r},\mathrm{t})\ . \qquad (49)$$

$H^{(\pm)}$ is the Dirac hamiltonian operator given by:

$$H^{(\pm)}=\mathrm{c}\alpha\cdot(\mathbf{P}-q\mathbf{A})+q\Phi\pm\beta\mu\mathrm{c}^{2} \qquad (50)$$

The magnetic potential **A** can couple states with different indices "*j*" because of matrix *α*, and this is necessary to induce transitions among positive and negative mass states. The scalar potential Φ cannot do that, so it will be forgotten in the following without loss of generality.

In order to simplify the calculations, it is assumed that **A** is uniform (so, ∇×**A** = 0) where the particle is, and we admit that its direction in space is parallel to the linear momentum vector **p** of the particle, so that **A** = *A***n** where **n** = **p**/|**p**| and *A* is a time-independent scalar.

Suppose that potential **A** is applied only to particles having a mass +*μ* and a charge *q*. So, only $H^{(+)}$in (50) depends on **A** and $H^{(-)}=H_{o}^{(-)}$. As a result, only the field Ψ_{+} in (2) will be modified:

$$\Psi_{+}(\mathbf{r},\mathrm{t})=\int d^{3}\mathbf{p}\sum_{S}\left(\Phi_{+,S,\mathbf{p}}(\mathbf{r})\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})+\ \Phi_{-,S,\mathbf{p}}(\mathbf{r})\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})\right), \qquad (51)$$

Introducing (51) in (49) we find the equations of motion for the operators "a":

$$i\,\hbar\frac{\partial}{\partial t}\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})=\omega_{0}\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})+\omega_{1}\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})\ , \qquad (52)$$

$$i\,\hbar\frac{\partial}{\partial t}\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})=-\omega_{0}\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})+\omega_{1}\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})\ . \qquad (53)$$

Solutions are:

$$\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})=\mathrm{f}^{*}(\mathrm{t})\ \mathrm{a}_{+,S,\mathbf{p}}+\mathrm{g}^{*}(\mathrm{t})\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}} \qquad (54)$$

$$\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})=\mathrm{f}(\mathrm{t})\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}\ -\mathrm{g}(\mathrm{t})\ \mathrm{a}_{+,S,\mathbf{p}} \qquad (55)$$

where

$$f(t) = e^{-i\lambda t/\hbar} + 2i\left(\frac{\omega_o + \lambda}{2\lambda}\right)\sin(\lambda t/\hbar) \tag{56}$$

$$g(t) = i\left(\frac{\omega_1}{\lambda}\right)\sin(\lambda t/\hbar) \tag{57}$$

with the initial conditions: f(0) = 1 and g(0) = 0 and

$$\omega_o \equiv \varepsilon(\mathbf{p}) - cqA\sin(\varphi) \qquad \omega_1 \equiv -cqA\eta\cos(\varphi)\ , \tag{58}$$

$$\lambda = \sqrt{\omega_o^2 + \omega_1^2} = \sqrt{(c(\mathbf{p} - qA\mathbf{n}))^2 + (\mu c^2)^2}\ . \tag{59}$$

5.2. Probability of Mass Inversion

As an example, consider the case of a particle of mass $+\mu$ and electric charge q = -e in the vacuum$|\,0\rangle_a$. The probability that, having created this particle in the state (S, **p**) at t = 0, we can recover it in the same state at t > 0 is:

$$|_a\langle 0|a_{+,S,\mathbf{p}}(t)a^\dagger_{+,S,\mathbf{p}}(0)|0\rangle_a|^2 = f^*(t)f(t) = 1 - G(t)\ , \tag{60}$$

where

$$G(t) \equiv \left(\frac{\omega_1}{\lambda}\right)^2 \sin^2(\lambda t/\hbar)\ . \tag{61}$$

When A = 0 this probability is always equal to 1. But when $A\neq 0$ the probability can become null. Indeed, there is a quantum resonance which appears when$(\omega_1/\lambda)^2$= 1 in (61). This happens if ω_o = 0 which implies from (58):

$$qA = |\mathbf{p}|\left[1 + \left(\frac{\mu c^2}{c|\mathbf{p}|}\right)^2\right]\ . \tag{62}$$

For q = -e we must have A < 0, and so **A** is opposed to **p**. Therefore, at specific times and this, periodically, it is not possible to recover the particle because G(t) = 1. Suppose t = t_o is one of those specific times. What is the probability at this moment that the mass of the particle is reversed? Using (45) and (54) one can show that:

$$|_b\langle 0|b_{j,S',\mathbf{p}'}(t)a^\dagger_{+,S,\mathbf{p}}(t)|0\rangle_b|^2 = \left(1 - G(t)\right)\cos^2(\varphi) + G(t)\sin^2(\varphi) \\ -2\eta\left(\frac{\omega_0}{\omega_1}\right)G(t)\sin(\varphi)\cos(\varphi) \tag{63}$$

only if $j = -$ (i.e. negative charge for "b" operators, see eq.(47)), $\mathbf{p}' = \mathbf{p}$ and $S' = S$, otherwise it is zero and:

$$\mathrm{b}_{j,S,\mathbf{p}}(\mathrm{t}) = \mathrm{e}^{-\mathrm{i}\left(j\varepsilon(\mathbf{p})\mathrm{t}/\hbar\right)}\ \mathrm{b}_{j,S,\mathbf{p}}\ . \tag{64}$$

Equation (63) gives the probability that having created a particle of mass $+\mu$ and electric charge -e in a state (S, $\mathbf{p}$) at time t in a vacuum$|\,0\rangle_{\mathrm{b}}$ we can recover it at the same moment in the same state with same electric charge, but with a mass $-\mu$. Note that mass inversion does not reverse the electric charge. Then at $\mathrm{t} = \mathrm{t_o}$, $G(\mathrm{t_o}) = 1$ and the probability is:

$$|_{\mathrm{b}}\langle 0|\mathrm{b}_{j,S',\mathbf{p}'}(\mathrm{t_o})\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(\mathrm{t_o})|0\rangle_{\mathrm{b}}|^2 = \sin^2(\varphi) = \left(\frac{\mathrm{c}|\mathbf{p}|}{\varepsilon(\mathbf{p})}\right)^2 . \tag{65}$$

The more the particle is relativistic, the more the probability approaches 1.

What we have done with a particle of mass $+\mu$ (i.e. $+\mu \to -\mu$) and electric charge -e, we can do it again with a mass $-\mu$ (i.e. $-\mu \to +\mu$) and electric charge -e (or +e) with a potential applied only to states of negative mass. The results are similar. This last situation corresponds to the case of primordial antiparticles which, under the action of an electromagnetic potential, would be "transferred" from Sakharov's "twin universe" to ours. Note that in 2016, the AMS-02 particle detector installed on the International Space Station measured an excess of antiprotons. Their energies were ranging from 10 to 20 GeV. Cholis et al. [19] have considered the possibility that such an excess would be caused by the annihilation of massive particles of dark matter (48-67 GeV) consistent with a gamma-ray excess of several GeV observed by Fermi-LAT in the center of the galaxy in 2011. After annihilation of such massive particles, there would be quark-antiquark pair production and finally the formation of antiprotons. However, it is worth noting that the energy of these antiprotons is large enough so that the probability of mass inversion in (65) is very close to 1. In this case, the value of potential A would be in the range of 30 to 60 Tesla·m.

6. Conclusion

In this work we have obtained, within the framework of Dirac's QFT, negative energy fields for fermions of spin ½. We have also shown that a connection between positive and negative energy fields exists, and that electric charges associated with these fields are opposite. As a result of applying an electromagnetic potential, we have highlighted how it could transfer a fermion with charge q, helicity S and linear momentum $\mathbf{p}$ in a mass state $+\mu$, into a fermion with same charge, helicity and linear momentum, but in a mass state -μ. Such a transfer is of course possible in the other direction too, if an electromagnetic potential is applied to the electric charge of a particle of mass $-\mu$.

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