International Journal of Modern Theoretical Physics, 2023, 12(1): 9-22

International Journal of Modern Theoretical Physics **ISSN: 2169-7426**

Journal homepage:www.ModernScientificPress.com/Journals/ijmtp.aspx *Florida, USA*

Article

Quantum Equations: Their Some Applications, and Possible Developments

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Article history: Received 25 January 2023, Revised 20 May 2023, Accepted 8 July 2023, Published 7 September 2023.

Abstract: First, Schrödinger equation and its some applications in DNA, etc., are studied. Second, we research the energy-momentum representation and basic equations, and which are applied to the lifetimes formulas of hyperons. Third, we search possible unified equations of fermions and bosons, both correspond to j+(1/2) order Bessel equation in spherical coordinates, and m order Bessel equation in column coordinates. Finally, we discuss possible 16 type developments of quantum equations.

Keywords: quantum equation, application, lifetime, hadron, Bessel equation.

1. Introduction

Quantum equation is an important base of quantum mechanics, and has many applications. But, Weinberg's conclusion is that there is no quantum mechanical interpretation of no serious shortcomings [1].

Recently, Nobre, et al., studied nonlinear relativistic and quantum equations with a common type of solution [2]. Giovannetti and Palma discussed master equations for correlated quantum channels [3]. Sarovar, et al., searched reduced equations of motion for quantum systems driven by diffusive Markov processes [4]. Arrigoni, et al., discussed nonequilibrium dynamical mean-field theory as an auxiliary quantum master equation approach [5]. Cai, et al., searched experimental quantum computing to solve systems of linear equations [6]. Koch and Frankcombe discussed basis expansion leaping as a new method to solve the time-dependent Schrödinger equation for molecular quantum dynamics [7]. Bassi, et al., researched uniqueness of the equation for quantum state vector collapse [8]. Zheng, et al., solved systems of linear equations with a superconducting quantum processor [9]. Pedernales, et al., discussed Dirac equation in (1+1)-dimensional curved spacetime and the multiphoton quantum Rabi model [10]. Balog, et al., searched disorder-driven quantum transition in relativistic semimetals, in which functional renormalization via the porous medium equation [11]. Can, et al., proposed spectral gaps and midgap states in random quantum master equations [12]. Bustamante, et al., discussed dissipative equation of motion for electromagnetic radiation in quantum dynamics [13]. Jiang, et al., discussed quantum simulation of the two-dimensional Weyl equation in a magnetic field [14]. Jäger, et al., searched Lindblad master equations for quantum systems coupled to dissipative bosonic modes [15]. In this paper, we discuss some applications of quantum equations, and their possible developments.

2. Schrödinger Equation and Its Applications

It is well-known that the basic equation of quantum mechanics is Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t}=(-\frac{\hbar^2}{2m}\nabla^2+U)\psi\,. \qquad (1)$$

The time-independent Schrödinger equation is:

$$\Delta\psi+\frac{2m}{\hbar^2}(E-V)\psi=0\,. \qquad (2)$$

Let $\frac{2m}{\hbar^2}(E-V)=k^2$, Eq.(2) becomes to the Helmholtz equation, i.e., wave equation:

$$\nabla^2\psi+k^2\psi=0\,. \qquad (3)$$

When V=E, it is Laplace equation. For the extensive quantum theory [16-19] only $\hbar\to H$.

Based on the extensive quantum mechanics in biology [20], Schrödinger equation at column coordinates and its solution may derive the double helical structure of DNA. It is necessity mathematical conclusion that quantum mechanics has symmetry [21-23].

Further, we researched the nonlinear biomechanics, which is related to chaos, fractal and soliton, etc [24,23]. An important character of the nonlinear biosystems is the formation of self-organization, which should decrease entropy. Complex biology provides a wide region for entropy decrease in various isolated systems. Moreover, we proposed the preliminary epidemic equations of COVID-19, and discuss their meaning [23].

Klein-Gordon equation is:

$$[p^2+m^2-(E-V)^2]\psi=0\,, \qquad (4)$$

For Coulomb potential $V=-(Z\alpha)/r$, Eq.(4) obtains:

$$[-\frac{d^2}{dr^2}+m^2-E^2+\frac{l'(l'+1)}{r^2}-2E\frac{Z\alpha}{r}]u=0. \qquad (5)$$

Total energy level is:

$$E(n,l)=\frac{m}{\sqrt{1+\frac{(Z\alpha)^2}{(n+\frac{1}{2}+\sqrt{(l+\frac{1}{2})^2-(Z\alpha)^2})^2}}}. \qquad (6)$$

For Schrödinger equation

$$[-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}+V(r)]u(r)=E(l)u(r). \qquad (7)$$

It may derive the Regge pole [25]. The energy becomes a complex number, and the real positive energy corresponds to the complex angular momentum [26].

General bound states [27]

$$(-\frac{1}{2}\nabla^2+W)\psi=E\psi. \qquad (8)$$

The potential $W(r)\approx-\frac{4k\sin(2kr)}{r}$. This is the waveform. The energy is $E=\frac{1}{2}k^2$.

3. Basic Equation of Energy-Momentum Representation and Lifetimes Formulas

It is known that momentum-energy operators in quantum mechanics are:

$$p_\alpha=-i\hbar\frac{\partial}{\partial x_\alpha}, E=i\hbar\frac{\partial}{\partial t}. \qquad (9)$$

The four-dimensional equations are:

$$p_\mu\psi=-i\hbar\frac{\partial}{\partial x_\mu}\psi. \qquad (10)$$

In momentum-energy operators representation there are the quantum equations [28,27]:

$$x_\mu\psi=i\hbar\frac{\partial}{\partial p_\mu}\psi. \qquad (11)$$

They include the quantum equation in energy representation:

$$T\psi=-i\hbar\frac{\partial}{\partial E}\psi. \qquad (12)$$

And the space operator equation [29]:

$$r\psi = i\hbar \frac{\partial}{\partial p}\psi . \quad (13)$$

Energy and time are two conjugate variables.

Further, Eq.(4) may extensive to:

$$i\hbar \frac{\partial}{\partial E}\psi = -(T+V)\psi . \quad (14)$$

This may obtain the lifetime formulas. When potential is rotation or at spherical coordinates, we may obtain $\frac{l(l+1)}{r^2}$. Such the equation with the spherical potential in bound states is:

$$\frac{d^2}{dr^2}\psi - \frac{l(l+1)}{r^2}\psi + \frac{2m}{\hbar^2}[E-V(r)]\psi = 0 . \quad (15)$$

The total energy is:

$$E = E_0 + An + Bl(l+1) . \quad (16)$$

This is similar to diatomic molecules.

Eq.(15) corresponds to equation in the spherical potential:

$$[\frac{d^2}{dp^2} - \frac{l(l+1)}{p^2} + \frac{E-V}{\hbar^2}]\psi = 0 . \quad (17)$$

$$\frac{d^2}{dE^2}\psi - \frac{l(l+1)}{E^2}\psi + [T(E)-V]\psi = 0 . \quad (18)$$

This adds the harmonic oscillator and the anharmonic oscillator $V = aE^2 + bE^3$, and $\tau = h/E$. In momentum representation, Eq.(15) becomes to:

$$[\frac{d^2}{dp^2} - \frac{l(l+1)}{p^2} + \frac{1}{\hbar^2}(E - ap^2 - bp^3)]\psi = 0 . \quad (19)$$

Planck formula was originally based on a series of harmonic oscillators. This should obtain different quantum lifetime.

We proposed the quantum equations of general relativity [29], for example:

$$g_{\mu\nu} \frac{\partial^2}{\partial p_\mu \partial p_\nu}\psi + \frac{s^2}{\hbar^2}\psi = 0 . \quad (20)$$

This is the form of combining quantum mechanics and general relativity, special relativity, and is the simplest unity between relativity and quantum [29].

From Eqs.(18)(19) we apply a complete analogy to derive the lifetime formula of the GMO form. They should describe the square of lifetime, and may derive a formula:

$$\tau^2 = \tau_0^2 + Al(l+1) + Bn + Cn^2 . \quad (21)$$

Let *l* corresponds to U-spin [30,31] and *n* corresponds to charge Q, Eq.(18) becomes the lifetime formula of hyperons:

$$\tau^2 = A[1+4U(U+1)-Q]-BQ^2. \qquad (22)$$

Eqs.(21)(22) are the symmetrical with the known GMO mass formula of hadrons [30,31]:

$$M = M_0 + AY + BI(I+1) + CY^2. \qquad (23)$$

It corresponds to the width formula:

$$\Gamma = \Gamma_0 + AY + BI(I+1) + CY^2. \qquad (24)$$

From this we obtain a lifetime relation of hyperons:

$$\tau^2(\Xi^0) + \tau^2(\Sigma^+)(9.05) = \tau^2(\Lambda) + \tau^2(\Sigma^-)(9.107). \quad (25)$$

Their agreements are better [29].

The usual lifetime formulas often are the empirical formulas for purely phenomenology. Now the improved theory must be the lifetime square, and the results are more consistent. It is amazing.

The original GMO mass formula is mass for baryon, and square of mass for meson. Here the lifetime formula is square of lifetime for baryon, and lifetime for meson.

For heavy flavor hadrons we proposed their lifetime formulas [32]. For Ξ_c^+ (usc), Λ_c^+ (udc), Ξ_c^0 (dsc) and Ω_c^0 (ssc), it is:

$$\tau = [1.4(2I-C)-S+3.4Q]\times10^{-13}. \qquad (26)$$

Then $\tau(\Xi_c^+)$=4.4, $\tau(\Lambda_c^+)$=2, $\tau(\Xi_c^0)$=1 and $\tau(\Omega_c^0)$=0.6, and the experimental data are (4.42 ± 0.26), (2.00 ± 0.06), (1.12 ± 0.13) and $(0.69\pm0.12)\times10^{-13}$ [33]. They all agree within the range of error. Further, for $\tau(\Xi_{cc}^{++}, ucc)$ =2.6 by this formula (26), it agrees accurately with the experimental data $(2.56\pm0.37)\times10^{-13}$ [34]. Form (26) we may predict $\tau(\Sigma_c^{++}$,uuc)=8.2, $\tau(\Sigma_c^+$,udc)=4.8 and $\tau(\Sigma_c^0$,ddc)=1.4.

Otherwise, the lifetime formula is extended for the mass formula. N(p,n), m(Λ_c^+)=2286, m(Σ_c)=2456, m(Ξ_c^0,Ξ_c^+)=2470, add m(Ω_c^0)=2695 are 7 particles, or add $m(\Xi_{cc})=3621$ are 8 particles. Let

$$m = M + AS + B[I(I+1) - S^2/2]. \qquad (27)$$

M=2286, A=-143, B=85, so m(Λ_c^+)=2286, m(Σ_c)=2456, m(Ξ_c^0,Ξ_c^+)=2470, but m(Ω_c^0)=2531, is not agree. Eq.(27) becomes

$$m = M - AS + BI(I+1). \qquad (28)$$

A=120, result is the same. If $M = M_0 + DC$, D=424, $m(\Xi_{cc}) = 3622$.

m(Λ_b^0)=5620, m(Σ_b)=5813, m(Ξ_b^-,Ξ_b^0)=5794, and m(Ω_b^-)=6046. For (28), M=5620, A=102, B=96, so m(Λ_b^0)=5620, m(Σ_b)=5812, m(Ξ_b^-,Ξ_b^0)=5794, but, m(Ω_b^-)=5824 is not agree. This lifetime formulas do not include Σ_c and Σ_b.

For (28) let B=0, M=1868=m(D), A=100, m(D_s)=1968.

For m=M+AS+BC, let M=5279=m(B), A=-88, B=996, m(B_s)=5367, m(B_c)=6275. Or baryons add N(p,n), and mesons add $\pi^{\pm,0}$.

Mass and lifetime may be unified as anharmonic oscillators in the spherical coordinate system, or emergent strings with rotations. Oscillation number n corresponds to Y and charge Q, and rotation number I corresponds to I and U [31].

It may be simply classified as the harmonic oscillators of various quantum numbers.

$\tau(\Omega^-)=0.82\times10^{-10}$ sec and $\tau(K_s^0)=0.895\times10^{-10}$ sec should obey the same lifetime formula of hyperons and mesons [30,31]

$$\tau = A[2U(U+1)-Q/2], \qquad (29)$$

and

$$\tau = A'[1+4U(U+1)-Q-2Q^2/3]. \qquad (30)$$

$$\tau = \tau_0 + A'[2U(U+1)-Q/2]+B'Q^2. \qquad (31)$$

$$\tau = A'[1+4U(U+1)-Q]-B'Q^2+C'Y(1+2Q). \qquad (32)$$

By Eq.(29), for Ω^-, let U=0, A=1.64. For K_s^0, let U=1, A=0.224.

4. Spin and Possible Unified Theory of Fermions and Bosons

It is known that fermion-pairs can form bosons. Further, any bosons can be composed of fermion-pairs. Therefore, fermions are more fundamental matter units, and bosons and fields can be composed of fermion-pairs. Such as p, e constitute hydrogen atoms, quark and antiquark pairs form mesons.

For the 3-dimensional central potential, let $\psi(r)=Y_{lm}(\Omega)u_{n,l}(r)/r$, Schrödinger equation is simplified to:

$$\{\frac{d^2}{dr^2}+\frac{2m}{\hbar^2}[E(n,l)-V(r)]-\frac{l(l+1)}{r^2}\}u_{n,l}(r)=0. \qquad (33)$$

The angular momentum quantum number *l* naturally produces a rotational potential.

The spin as part of the total angular momentum shows as the electron spin s is split to 2 in the magnetic field, which should develop Pauli equation, and in Eq.(3):

$$k^2=\frac{2m}{\hbar^2}E\pm\frac{2e}{c\hbar}(l+\frac{1}{2}). \qquad (34)$$

For different the angular momentum M and potential V substitution into the same k^2, the separation variables are still the same equation.

Spin corresponds probably to different structures of electron [35], or propose new equations, such as the angular momentum equation [28]:

$$M\psi = -i\hbar \frac{\partial}{\partial \varphi}\psi . \quad (35)$$

In spherical coordinates, by general method let $\psi = R(r)Y(\theta,\varphi) = R(r)\Theta(\theta)\Phi(\varphi)$,

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + [x^2 - (j+\frac{1}{2})^2]y = 0 . \quad (36)$$

It is j+(1/2) order Bessel equation, which is also discussed by Weinberg [1].

In column coordinates, by general method let $\psi = R(\rho)Z(z)\Phi(\varphi)$.

$$x^2 \frac{d^2 R}{dx^2} + x\frac{dR}{dx} + [x^2 - m^2]R = 0 . \quad (37)$$

R(x) is m order Bessel equation. If quantized j+(1/2) and m are all spins, Eqs.(36) and (37) will correspond to fermions and bosons, respectively.

This is corresponding relations between Bessel equation and spin:

spherical coordinate	j	fermions	Dirac equation	FD statistics
column coordinate	m	bosons	K-G equation	BE statistics, and BEC

If both are related to topology, so fermions in the spherical coordinates are independent of each other, and corresponds to Pauli exclusion principle; bosons in the column coordinates are open, which may have multiple particles.

It is known that Kerr solution (1963) and Kerr-Newman solution (1965) have the column symmetry. Boyer-Lindquist (1967) and Carter (1968) completed Kerr solution and the maximum expansion of Kerr-Newman solution. This should correspond to my plane solution of general relativity [36]. If this is related to bosons, so it will be unstable. For evolution, it corresponds to the DNA. We should research the relations of DNA and bosons.

The Wheeler-De Witt functional differential equations with infinite degrees of freedom cannot almost be solved [37]. The covariant quantization eventually abandoned the point particle model, and developed into string theory. The loop quantum gravity is found along the canonical quantization path.

In general relativity, Schwarzschild solution and Reissner-Nordstrom solution have the spherical symmetry. It is relation to fermions. Except photon (s=1), only three fermions p, e, v are stable.

For quantum gravity theory, quantum particles are the product of gravitational fields in Hadley's theory [38,39]. Quantum gravity is namely quantized graviton [40-42].

Einstein thought that space and others all originate from the field, which corresponds to the bosons. While bosons can be produced by fermions, and formed in pairs (as photon γ =vv), so there are still two aspects. Most basic graviton (spin s=2), photon γ (s=1), for strong and weak interactions (except π and K, s=0) gluon and W-Z are all s=1, which should be basis of field theory for the great unification of strong and weak electromagnetic forces.

Strong and weak interactions as short-range should be unified. Except different action ranges their main character is: strong interactions are attraction each other, and weak interactions are mutual repulsion and derive decay. We proposed a new method on their unification, whose coupling constants are negative and positive, respectively. Further, we researched a figure on the unification of the four basic interactions in three-dimensional space. So far the high energy experiments in the past sixty years have shown that the smallest mass fermions are proton, electron, neutrino and photon, which form the simplest model of particles. These fermions seem to be inseparable truth "atoms" (elements), because further experiments derive particles with bigger mass. They correspond to four interactions, and are also only stable particles. The final simplest theory is based on leptons (e-ν_e) and nucleons (p-n) or (u-d) in quark model with SU(2) symmetry and corresponding Yang-Mills field. Other particles and quark-lepton are their excited states. We discussed the simplest interactions and simplified QCD, and some possibly developed directions of particle physics, for example, violation of basic principles, and precision and systematization of the simplest model, etc [43].

The essence of the renormalization is to eliminate the infinite background, i.e., related to the vacuum and the infinite potential. These and the renormalization are not required in the gravitational field.

We should research the entangled field is what field? This is scalar, or vectors, tensor, spinor field, etc. It seems to be dual fields, inductive fields resonant to each other, and is the wave-field with the same frequency.

5. Some Possible Developments of Quantum Theory

It is known that baryon is Dirac equation, and meson is Klein-Gordon equation. Both are approximately Schrödinger equation. This may obtain $r_n = an^2$, and corresponding equations may derive $p_n = bn^2$, and $E_n = cn^2 = \frac{p^2}{2m}$, $p = Dn$, etc.

Weinberg proposed that we should take it seriously and find a more satisfying other possible theory, and quantum mechanics is only a good approximation of it [1]. Here we research some possible developments of quantum theory:

1). This should produce a similar potential of $c\bar{c}$ and $b\bar{b}$ [44,45]:

$$V=-\frac{a}{r}+br\,. \tag{38}$$

Here *a*=0.52, b=0.18GeV2.

2). Life changes with energy. There are vibrations, such as resonant or nonresonant, and rotation. It can be used to study life, its spectrum and quantized.

3). Space-time has an uncertainty relation, and is quantization. Combined with the logical structure of quantum mechanics [46], it may have statistics. p and r conjugate, E and t conjugate, M_z and φ conjugate. Three are symmetry, and form uncertainty relations each other. Here

$$i\hbar\frac{\partial}{\partial\varphi}\psi=M_z\psi\,,\ -i\hbar\frac{\partial}{\partial M_z}\psi=\varphi\psi\,. \tag{39}$$

The momentum-space equation and its solution of a stationary state are known. Its conjugate equation is:

$$r\psi=i\hbar\frac{\partial}{\partial p}\psi\,, \tag{40}$$

Its solution of a stationary state is:

$$\psi(p)=Ne^{-ipr/\hbar}\,. \tag{41}$$

For the energy-time equation, its conjugate equation is:

$$t\psi=-i\hbar\frac{\partial}{\partial E}\psi\,. \tag{42}$$

Its solution of a stationary state is:

$$\psi(E)=Ne^{iEt/\hbar}\,. \tag{43}$$

For the angular momentum-rotation angle equation [28], its conjugate equation is:

$$i\hbar\frac{\partial}{\partial M_z}\psi=\varphi\psi\,. \tag{44}$$

Its solution of a stationary state is:

$$\psi(M_z)=Ne^{-iM_z\varphi/\hbar}\,. \tag{45}$$

4). This method can be generalized to the mechanical wave theory [47], and obtain the conjugate equation, and the mass-lifetime formula, etc.

5). Unification for the three-generation quark equations. Only n, l and Y, Q are different, except u, d, the other I all is 0. There have relations with P, C, PC, and PCT.

6). The relativity equation is:

$$s^2=T^2-R^2, T^2=s^2+R^2\,. \tag{46}$$

$$T^2\psi = -\hbar^2 \frac{\partial^2}{\partial E^2}\psi = (s^2 + R^2)\psi . \qquad (47)$$

$$\frac{\partial^2}{\partial p_\mu^2}\psi = [\frac{\partial^2}{\partial p^2} - \frac{\partial^2}{\partial E^2}]\psi = \frac{s^2}{\hbar^2}\psi . \qquad (48)$$

It is similar Klein-Gordon equation. The similar Dirac equations are:

$$i\hbar \frac{\partial}{\partial E}\psi = (\alpha s + \beta R)\psi . \qquad (49)$$

$$\gamma_\mu \frac{\partial}{\partial p_\mu}\psi = s\psi . \qquad (50)$$

It is replaced by various space-time metric, and can obtain various equations of similar quantum mechanics, such as for the vacuum $s^2 = 0$.

7). Further, we should research equation and so on for anyon.

8). It is known that the charged Dirac equations become non-relativistic, so can obtain spin. By similar way some effect should be obtained. But this must first consider the change in space-time in the electromagnetic field, as for Reissner-Nordstrom metric

$$g_{00}c^2dt^2 = (1 - \frac{2m}{r} + \frac{4\pi GQ^2}{c^4r^2})c^2dt^2 . \qquad (51)$$

The equation with charge is:

$$(\gamma_\mu \frac{\partial}{\partial p_\mu} - s + AQ^n)\psi = 0 . \qquad (52)$$

Further, this should combine the electromagnetic general relativity [48].

Non-relativity is invariant T, and corresponds to the stationary state. For Eq.(4), its solution is:

$$\psi = A\exp(iTE/\hbar) . \qquad (53)$$

$$\sqrt{g_{00}}t\psi = -i\hbar\sqrt{g_{00}}\frac{\partial}{\partial E}\psi . \qquad (54)$$

9). When time has a direction, the corresponding equations may be developed. At this time the time may be the vector, and corresponds to the 3 dimension time [49]:

$$-\hbar^2\Delta\psi = \vec{t}\,\psi . \qquad (55)$$

It can introduce the spherical coordinates, and obtain $\frac{l(l+1)}{r^2}$.

10). This follows that space-time changes with energy, and consistent with general relativity. For general relativity, $s^2 = g_{\mu\nu}x_\mu x_\nu$,

$$x_\mu\psi = i\hbar \frac{\partial}{\partial p_\mu}\psi = (s^2 / g_{\mu\nu}x_\nu)\psi . \qquad (56)$$

General case is:

$$s^2\psi = g_{\mu\nu}x_\mu x_\nu\psi = -\hbar^2 g_{\mu\nu}\frac{\partial^2}{\partial p_\mu \partial p_\nu}\psi \ . \qquad (57)$$

There is the electromagnetic field, it is:

$$g_{\mu\nu}x_\mu x_\nu\psi + \hbar^2 g_{\mu\nu}\frac{\partial^2}{\partial p_\mu \partial p_\nu}\psi + A(Q^2+V)\psi = 0 \ .(58)$$

For special relativity,

$$s^2\psi = [(ct)^2 - r^2]\psi = -\hbar^2[\frac{\partial^2}{\partial (E/c)^2} - \frac{\partial^2}{\partial p^2}]\psi \ . \quad (59)$$

Various metrics, as Schwarzschild metric

$$(1-\frac{2m}{r})c^2dt^2 \ . \qquad (60)$$

$$T^2\psi = (1-\frac{2m}{r})(-\hbar^2\frac{\partial^2}{\partial E^2})\psi \ . \qquad (61)$$

$$(1-\frac{2m}{x_\mu})x_\mu^2\psi = (x_\mu^2 - 2mx_\mu)\psi = (-\hbar^2\frac{\partial^2}{\partial p_\mu^2} - 2im\hbar\frac{\partial}{\partial p_\mu})\psi \ . (62)$$

This equations have the first-order and second-order differential. Reissner-Nordstrom metric is $\frac{R^2(t)}{1-kr^2}dr^2$. $g_{\mu\nu}$ correspond to E and V.

We can further unify quantum theory and general relativity. Relativity focuses on space-time, while quantum theory focuses on matter and mass-energy. This can particularly describe the quantized discrete space-time, in which dx_μ is a quantum space-time. We proposed the general equations are [29]:

$$G_{\mu\nu}\psi = \kappa T_{\mu\nu}\psi \ . \qquad (63)$$

This is replaced by the operator representation of the energy-momentum and space-time. It can be related to the Wheeler-de Witt cosmic equations of quantum mechanics.

11). It can be similarly developed by method of nonlinear quantum theory [30,50], in which F corresponds to $g_{\mu\nu}$, and Γ corresponds to potential V.

12). In astronomy and cosmology, it corresponds to general relativity. Such various models of cosmic variation can be obtained, as the Big Bang, inflation, the circular universe, etc.

13). Charged particles may combine the cosmical electrodynamics [51].

14). Combining extensive quantum theory [16-20], the possible wave functions must also be generalized to extensive quantum theory.

15). Wendt studied systematically quantum mind and social science, and proposed that people are the walking wave functions based on the causal closure (or completeness) of physics (CCP) [52]. This research core is that quantum theory can explain the phenomena of consciousness and intentionality, and an important basis is the quantum entanglement and macroscopic entanglement. We discussed quantum sociology, whose bases must be the extensive quantum theory and the social individual-wave duality [53]. Quantum may be extended to people, animals and plants, all the life span, evolution, etc. V (E) is a factor affecting life, time and its structure, and is external factor and external potential. Infinite deep potential trap, i.e., any life is all limited, and different people have different energy levels. It may combine Maslow's different needs.

16). In statistical mechanics and entropy,

$$S\psi = \frac{E}{T}\psi = i\hbar \frac{1}{T}\frac{\partial}{\partial t}\psi \, . \qquad (64)$$

The simplest is h→h/T.

Based on the Noether theorem, energy E is related to time t: (1). E is conserved, t is uniform; E is constant, and t is also constant. (2). E change, t has direction; especially when dE/T=dS. (3). More generally, when various changes, t shows the direction. (4). E and various cycles, t also cycle, or spiral rise. (5). E slows, as does t and longevity. (6). When chaotic and nonlinear change, t is not uniform and has very critical moments.

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