International Journal of Modern Mathematical Sciences, 2019, 17(2): 151-168

Modern Scientific Press

International Journal of Modern Mathematical Sciences ISSN:2166-286X

Journal homepage: www.ModernScientificPress.com/Journals/ijmms.aspx *Florida, USA*

Article

A Purchasing Inventory Model for Fading Products with Non-escalating Demand under Stock-Induced Holding Cost with and without Shortage

R.P.Tripathi* and Sachin Mishra

Department of Applied Sciences and Humanities, KNIT, Sultanpur

*Author to whom correspondence should be addressed; E-Mail:tripathi_rp0231@rediffmail.com

Article history: Received 17 June 2019; Revised 19 August 2019; Accepted 20 August 2019; Published 23 September 2019.

Abstract: This paper presents an EOQ (Economic Order Quantity) model for spoilng commodities by means of non-increasing demand and stock-induced demand. Shortages are allowed in the second model. In this study, a purchasing inventory model for failing products with inventory linked demand is considerd and optimal solution is illustrated in higher organizes equation. Two inventory models are presented for different situations. In the first situation EOQ model is assumed with time induced deterioration and is the second situation deterioration with shortages. The models are developing for these two cases. Most advantageous inventory lot size that minimizes total cost is illustrated. Numerical examples are sensitivity analysis is also illustrated.

Keywords: Deterioration; Stock-linked holding cost; demand; production; inventory

Mathematics Subject Classification: 90B05

1. Introduction

Productivity depicts diverse parameters of the competence of product. The main basis of difference between a varieties of productivity dimension and or data availability. Production is a vital issue in production presentation of firm and countries. When multiple inputs are considered, the measure is said to be multi-factor productivity. If the efforts are labor capital, and the yields are value added middle outputs, the compute is called total factor productivity (TFP). TFP measure the remaining grown that cannot be clarified by the rate of transform in the services of labor and capital. What all outputs are included in the productivity measure it is said to be total productivity. The productivity development is an industry can be circulated in a number of several ways.

- to the work force through better wages and circumstance.
- to shareholders and superannuation funds through increased profits and extra distributions.
- to purchaser through lower prices.
- to the surroundings through more stringent environmental protection.
- to governments through increases in tax payments.

There are two categories of items available in the market- deteriorating items and non-deteriorating items. Deteriorating items again can be classified into two subcategory-time dependent deterioration and constant deterioration. Deteriorating items are deteriorated with time. In this paper two models are considered. In the first model deterioration is time dependent and in the second model it is constant. Demands for these commodities exist in the market for finite time and these kinds of demands are dependent on time.

1.1. Literature Review

Production is a natural phenomenon to suvive any where in the society, city, state and country. Yang and Lee [1] studied an economic production quantity (*EPQ*) model for a deteriorating item with finite production rate.Yan *et al*. [2] designed an *EPQ* model in view of the decline rate to be very small. Ghiami and Williams [3] presented an *EPQ* model for deteriorating commodities under two -echlon system. Lee and Kim [4] analyzed a single-seller, single-buyer supply chain for a deteriorating commodities assuming a probability for item be defective as a failure in the production processes. Ghare and Schrader [5] addressed an exponentially decomposing inventory for an invariable demand. Dye [6] established an inventory system by means of non-instantaneous deteriorating goods. Researcher including Abad [7] , Wee [8] , Tsao and Sheen [9], Wang and Li [10] , Das *et al,* Abad [7] , Wee [8] , Tsao and Sheen [9], Wang and Li [10] , Das *et al*. [11] , Taleizadeh *et al*. [12] developed EPQ models that focused on deterioration inventory model.

In traditional EOQ/EPQ model demand rate is considered as stable. While in actual performance demand rate fluctuates with time due to season, deterioration, unavailability, inflation etc. Dave and Patel [13] established an inventory model under time-dependent demand.Xu and Wang [14] explored an EOQ model for exponentially fading item through linearly time unstable demand and finite shortage cost. Sarkar *et al* .[15] introduced an inventory model in which demand rate is inventory linked. Ghosh and Chaudhuri [16] presented an EOQ model for a deteriorating commodity for a quadratic time-varying demand and shortage. Ghosh *et al*. [17] designed an optimal inventory replenishment policy for a failing item, time-quadratic demand and time-linked partial back logging which depends on the length of coming up time for next replenishment over a finite time horizon and variable replenishment cycle. Various examination have previously been made by numerous inventory modelers on EOQ/EPQ models for time-linked demand like Chen [18], Lee and Hsu [19], Hsich and Dye [20], Wee and Wang [21], Shah *et al*. [22]. Shukla *et al*. [23], Mahata [24], Tripathi *et al*. [25, 26, 27], Tripathi [28].

Nobil and Sedigh [29] developed an EPQ model for defective production system and uncertain upline : Alfares and Ghaithan [30] pointed out an EOQ and EPQ models with variable holding cost. Some of the notable researches in this direction are Tripathi [31] , Tripathi *et al*. [32] and Taleizadesh and Bayatloo [33] designed a model for pricing and order decisions.

In this study, a purchasing EOQ model for spoiling products by means of inventory-induced holding cost is considerd and optimal situation is obtained. Two inventory models are presented under two different policies. The first inventory policy covers the case that the model is adopted with inventory-dependent holding cost with time linked deteriorating and demand and in the second policy covers the case in model with similar holding cost and demand as in first case with constant deterioration under allowed shortages. The remainder part of the paper is framed as follows. In section 2,literatyre review is given. Section 3 presents the notations and assumption. In section 4 mathematical formulation optimal solution is developed. In section 5 & 6 numerical explanation and sensitivity investigation are discussed. Lastly, conclusion are illustrated in section 7.

2. Notations and Assumption

2.1. Notations

I(t): inventory level at time '*t*'

$D(t) = a - bt$: demand rate, a > 0, 0 < b < 1

C_0 : setup cost

$\theta(t) = \alpha + \beta t$: time dependent deterioration α >0, β > 0

C_d : deteriorating cost

T **:** cycle time

θ **:** deterioration rate from model II, $0 < \theta < 1$

Q : order quantity

T_1 : time to finite positive inventory for model II

C : total cost per cycle

Assumptions : Following assumption are used to build up the model.

(i). Demand rate is considered as time- sensitive.

(ii).Deterioration rate is time-linked for model 1 and constant for model II.

(ii).Planning horizon is finite.

(iv).Shortage is permitted and steady.

(v).Lead time is negligible.

(vi).Holding cost is stock-linked

3. Mathematical Formulation

3.1. Purchasing Inventory Model with Time-Induced Deterioration with non-increasing Time Sensitive Demand

In this case, model is developed for time – sensitive failing commodities in which demand rate is time- linked, where Q is the component of item of inventory system at opening of every sequence. Inventory level diminishes due to demand and deterioration till it turns out to be zero in [0, T]. Entirety process is recurring. Differential equation leading in the state is:

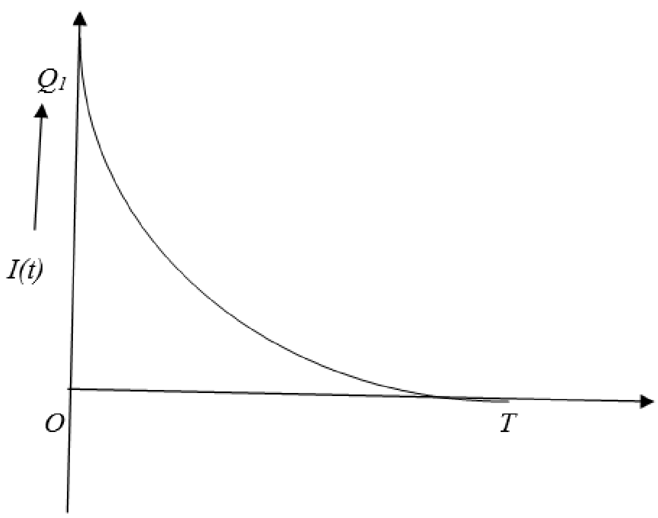

Fig. 1. Pictorial representation inventory level vs. time

$$\frac{dI(t)}{dt}+(\alpha+\beta t)I(t)=-(a-bt);0\le t\le T \tag{1}$$

With the boundary conditions $I(0)=Q_1;I(T)=0$ (2)

Solution of Eq. (1) under conditions (2) is:

$$I(t)=\left\{\frac{3\alpha(a\alpha-b)-3\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-3a\left(\alpha^2-\beta\right)}{6}\right\}t^3+\left\{\frac{48a\alpha+4\left(\alpha^2-\beta\right)\left(a\alpha^2+a\beta-2b\alpha\right)}{48}\right.$$
$$\left.+\frac{12(a\alpha-b)\left(\alpha^2-\beta\right)T^2+24a\left(\alpha^2-\beta\right)T-24(a\alpha-b)-3b\left(\alpha^4-\beta^2\right)}{48}\right\}t^2$$
$$+\left\{\frac{3b\alpha\left(\alpha^2+\beta\right)T^4-24a-4\alpha\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3-12\alpha(a\alpha-b)T^2-24a\alpha T}{24}\right\}t$$
$$+\left\{\frac{4\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3+12(a\alpha-b)T^2+24aT-3b\left(\alpha^2+\beta\right)T^4}{24}\right\} \tag{3}$$

and, order quantity is:

$$Q=I(0)=\left\{\frac{4\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3+12(a\alpha-b)T^2+24aT-3b\left(\alpha^2+\beta\right)T^4}{24}\right\} \tag{4}$$

Total cost: The total cost contains the addition of set up cost, holding cost and deteriorating cost

(i).Setup cost (SU) = $\frac{C_0}{T}$ (5)

(ii).Holding cost(HC) = $\frac{1}{T}\int_0^T hI(t)dt$

$$=h\left[\left\{\frac{3\alpha(a\alpha-b)-3\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-3a\left(\alpha^2-\beta\right)}{24}\right\}T^3+\left\{\frac{48\alpha a+4\left(\alpha^2-\beta\right)\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)}{144}\right.\right.$$
$$\left.+\frac{12(a\alpha-b)\left(\alpha^2-\beta\right)T^2+24a\left(\alpha^2-\beta\right)T-24(a\alpha-b)-3b\left(\alpha^4-\beta^2\right)T^4}{144}\right\}T^2$$
$$+\left\{\frac{3b\alpha\left(\alpha^2+\beta\right)T^4-24a-4\alpha\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3-12\alpha(a\alpha-b)T^2-24a\alpha T}{48}\right\}T$$
$$\left.+\left\{\frac{4\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3+12(a\alpha-b)T^2+24aT-3b\left(\alpha^2+\beta\right)T^4}{24}\right\}\right] \tag{6}$$

(iii). Deteriorating cost (*DC*) $=\frac{C_d}{T}\int_0^T(\alpha+\beta t)C_d I(t)dt$

$$=\alpha C_d\left[\left[\left\{\frac{3\alpha(a\alpha-b)-3\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-3a\left(\alpha^2-\beta\right)}{24}\right\}T^3+\left\{\frac{48\alpha a+4\left(\alpha^2-\beta\right)\left(a\alpha^2-a\beta-2b\alpha\right)}{144}\right.\right.\right.$$

$$\left.+\frac{12(a\alpha-b)\left(\alpha^2-\beta\right)T^2+24a\left(\alpha^2-\beta\right)T-24(a\alpha-b)-3b\left(\alpha^4-\beta^2\right)T^4}{144}\right\}T^2$$

$$+\left\{\frac{3b\alpha\left(\alpha^2+\beta\right)T^4-24a-4\alpha\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3-12\alpha(a\alpha-b)T^2-24a\alpha T}{48}\right\}T$$

$$\left.\left.+\left\{\frac{4\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3+12(a\alpha-b)T^2+24aT-3b\left(\alpha^2+\beta\right)T^4}{24}\right\}\right]\right]+$$

$$\beta C_d\left[\left[\left\{\frac{3\alpha(a\alpha-b)-3\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-3a\left(\alpha^2-\beta\right)}{30}\right\}T^4+\left\{\frac{48\alpha a+4\left(\alpha^2-\beta\right)\left(a\alpha^2-a\beta-2b\alpha\right)}{192}\right.\right.\right.$$

$$\left.+\frac{12(a\alpha-b)\left(\alpha^2-\beta\right)T^2+24a\left(\alpha^2-\beta\right)T-24(a\alpha-b)-3b\left(\alpha^4-\beta^2\right)T^4}{192}\right\}T^3$$

$$+\left\{\frac{3b\alpha\left(\alpha^2+\beta\right)T^4-24a-4\alpha\left\{a\left(\alpha^2+\beta\right)-2b\alpha\right\}T^3-12\alpha(a\alpha-b)T^2-24a\alpha T}{72}\right\}T^2$$

$$\left.+\left\{\frac{4\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)T^3+12(a\alpha-b)T^2+24aT-3b\left(\alpha^2+\beta\right)T^4}{48}\right\}T\right] \quad (7)$$

Note: Higher powers of *T* are neglected.

Therefore Total cost = *SU* + *HC* + *DC*

$$C=\frac{C_0}{T}+\frac{(\alpha+\beta)(h+C_d)}{72}\Big[108aT+2\Big\{24\alpha a+2\left(\alpha^2-\beta\right)\left(a\left(\alpha^2-\beta\right)-2b\alpha\right)-12(a\alpha-b)-12a+$$
$$45(a\alpha-b)\Big\}T^2+3\Big\{6\alpha(a\alpha-b)-6\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-6a\left(\alpha^2+\beta\right)+8a\left(\alpha^2-\beta\right)-$$
$$b\left(\alpha^4-\beta^2\right)-96\alpha+6(a\alpha-b)+10\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)\Big\}T^3\Big] \quad (8)$$

$$\frac{dC}{dT}=-\frac{C_0}{T^2}+\frac{(\alpha+\beta)(h+C_d)}{72}\Big[108a+4\Big\{24\alpha a+2\left(\alpha^2-\beta\right)\left(a\left(\alpha^2-\beta\right)-2b\alpha\right)-12(a\alpha-b)-$$
$$12a+45(a\alpha-b)\Big\}T+9\Big\{6\alpha(a\alpha-b)-6\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-6a\left(\alpha^2+\beta\right)+8a\left(\alpha^2-\beta\right)+$$
$$b\left(\alpha^4-\beta^2\right)-96\alpha+6(a\alpha-b)+10\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)\Big\}T^2\Big] \quad (9)$$

$$\frac{d^2C}{dT^2}=\frac{2C_0}{T^3}+\frac{(\alpha+\beta)(h+C_d)}{72}\Big[4\Big\{24\alpha a+2\left(\alpha^2-\beta\right)\left(a\left(\alpha^2-\beta\right)-2b\alpha\right)-12(a\alpha-b)-$$
$$12a+45(a\alpha-b)\Big\}+18\Big\{6\alpha(a\alpha-b)-6\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-6a\left(\alpha^2+\beta\right)+8a\left(\alpha^2-\beta\right)+$$

$$b\left(\alpha^4-\beta^2\right)-96\alpha+6\left(a\alpha-b\right)+10\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)\Big\}T\Big] \tag{10}$$

For extremum putting $\frac{dC}{dT}=0$, we get

$$\frac{C_0}{T^2}-\frac{(\alpha+\beta)(h+C_d)}{72}\Big[108a+4\Big\{24\alpha a+2\left(\alpha^2-\beta\right)\left(a\left(\alpha^2-\beta\right)-2b\alpha\right)-12\left(a\alpha-b\right)-12a+45\left(a\alpha-b\right)\Big\}T$$
$$+9\Big\{6\alpha\left(a\alpha-b\right)-6\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)-6a\left(\alpha^2+\beta\right)+8a\left(\alpha^2-\beta\right)+b\left(\alpha^4-\beta^2\right)-96\alpha+6\left(a\alpha-b\right)$$
$$+10\left(a\left(\alpha^2+\beta\right)-2b\alpha\right)\Big\}T^2\Big]=0 \tag{11}$$

4.2. Purchasing Inventory Model for Deteriorating Item with Shortages

This model is established for fading item in which shortages are permitted and demand rate is time- sensitive. Inventory level diminishes due to demand and deterioration till it becomes zero in during [0 ,T_1]. Shortages starts just after T_1 and finished at T. The total process is repeated. Inventory at any instant is shown in Fig. 2.

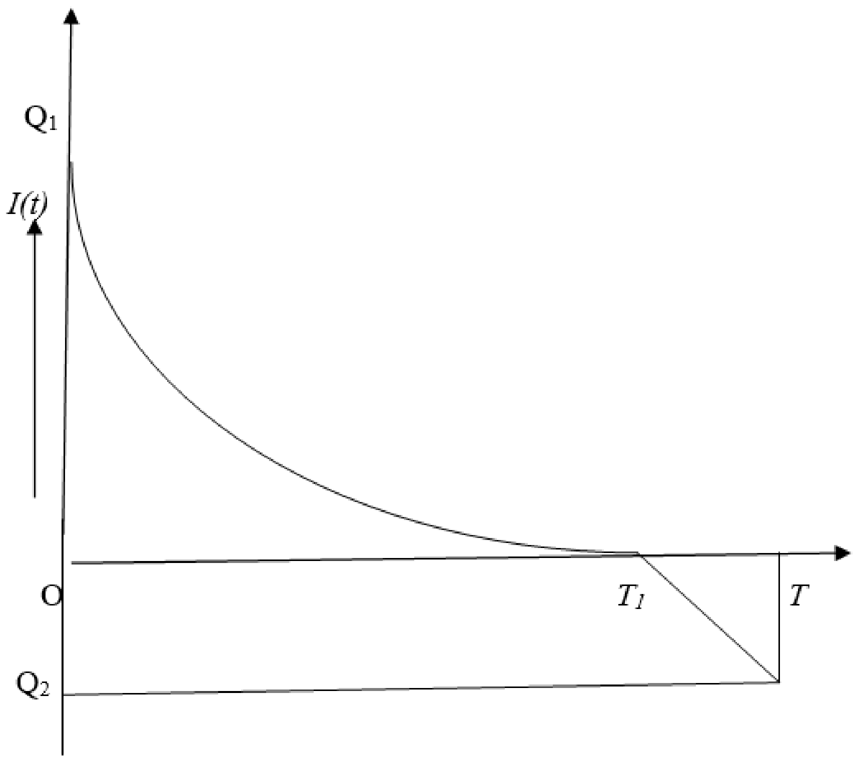

Fig. 2. Pictorial representation inventory level vs. time under shortages

$$\frac{dI(t)}{dt}+\theta I(t)=-\left(a-bt\right);0\le t\le T_1 \tag{12}$$

$$\frac{dI(t)}{dt}=-a;T_1\le t\le T \tag{13}$$

with the boundary conditions $I(0)=Q; I(T_1)=0$ (14)

Solution of (12) and (13) are:

From Eq.(12) and (13);we get

$$I(t)=\left[\left\{\frac{6a+3(a\theta-b)T_1+(a\theta-b)T_1^2}{6}\right\}T_1-\left\{\frac{6a+6a\theta T_1+3\theta(a\theta-b)T_1^2+\theta(a\theta-b)T_1^3}{6}\right\}t+\right.$$

$$\left\{\frac{6(a\theta+b)+6a\theta^2T_1+3(a\theta-b)\theta^2T_1^2+(a\theta-b)\theta^2T_1^3}{12}\right\}t^2+$$

$$\left.\left\{\frac{6\theta(a\theta+b)+6\theta^3T_1+3(a\theta-b)\theta^3T_1^2+(a\theta-b)\theta^3T_1^3}{36}\right\}t^3\right] \tag{15}$$

and $I(t)=-a(t-T_1)$, respectively (16)

From Eq.(15) and (16), we get

$$I(0)=Q_1=\left\{\frac{6aT_1+3(a\theta-b)T_1^2+(a\theta-b)T_1^3}{6}\right\} \tag{17}$$

$$I(T)=Q_2=a(T-T_1)$$

$$Q=Q_1+Q_2=\left\{\frac{(a\theta-b)T_1^3+3(a\theta-b)T_1^2+6aT}{6}\right\} \tag{18}$$

Total cost: The total cost contain the sum of the set up cost, holding cost and deteriorating cost.

(i).Setup cost = $\frac{C_0}{T}$ (19)

$$\text{(ii).}HC=\frac{1}{T}\int_0^{T_1}hI(t)dt=\frac{h}{T}\left[\left\{\frac{6a-6bT_1(a\theta-b)(2-3\theta)T_1^2-\theta(a\theta-b)T_1^3}{12}\right\}T_1^2+\right.$$

$$\left\{\frac{6(a\theta+b)+6a\theta^2T_1+3(a\theta-b)\theta^2T_1^2+(a\theta-b)\theta^2T_1^3}{36}\right\}T_1^3+$$

$$\left.\left\{\frac{6\theta(a\theta+b)+6\theta^3T_1+3(a\theta-b)\theta^3T_1^2+(a\theta-b)\theta^3T_1^3}{144}\right\}T_1^4\right] \tag{20}$$

$$\text{(iii).}DC=\frac{\theta C_d}{T}\int_0^{T_1}I(t)dt=\frac{\theta C_d}{T}\left[\left\{\frac{6a-6bT_1(a\theta-b)(2-3\theta)T_1^2-\theta(a\theta-b)T_1^3}{12}\right\}T_1^2+\right.$$

$$\left\{\frac{6(a\theta+b)+6a\theta^2T_1+3(a\theta-b)\theta^2T_1^2+(a\theta-b)\theta^2T_1^3}{36}\right\}T_1^3+$$

$$\left.\left\{\frac{6\theta(a\theta+b)+6\theta^3T_1+3(a\theta-b)\theta^3T_1^2+(a\theta-b)\theta^3T_1^3}{144}\right\}T_1^4\right] \tag{21}$$

(iv). $SC=\frac{C_s}{T}\int_{T_1}^{T}-a\left(t-T_1\right)dt=\frac{aC_s\left(T-T_1\right)^2}{2T}$

Therefore Total cost= $SU+HC+DC+$

$$SCC=\frac{C_0}{T}+\frac{\left(h+\theta C_d\right)}{24T}\Big[12aT_1^2+4T_1^3\left(a\theta-2b\right)+\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^4+$$

$$\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^5\Big]+\frac{aC_s\left(T-T_1\right)^2}{2T} \quad (22)$$

Differentiating (22) w.r.t. T_1 and T , twice .we get

$$\frac{\partial C}{\partial T_1}=\frac{\left(h+\theta C_d\right)}{24T}\Big[24aT_1+12T_1^2\left(a\theta-2b\right)+4\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^3+$$

$$5\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^4\Big]-\frac{aC_s\left(T-T_1\right)}{T} \quad (23)$$

$$\frac{\partial C}{\partial T}=-\frac{C_0}{T^2}-\frac{\left(h+\theta C_d\right)}{24T^2}\Big[24aT_1+12T_1^2\left(a\theta-2b\right)+4\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^3+$$

$$5\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^4\Big]+\frac{aC_s}{2T^2}\left(T^2-T_1^2\right) \quad (24)$$

$$\frac{\partial^2 C}{\partial T_1^2}=\frac{\left(h+\theta C_d\right)}{6T}\Big[6a+6T_1\left(a\theta-2b\right)+3b\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^2+$$

$$5\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^3\Big]+\frac{aC_s}{T} \quad (25)$$

$$\frac{\partial^2 C}{\partial T^2}=\frac{2C_0}{T^3}-\frac{\left(h+\theta C_d\right)}{12T^3}\Big[24aT_1+12T_1^2\left(a\theta-2b\right)+4\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^3+$$

$$5\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^4\Big]+\frac{aC_sT_1^2}{T^3} \quad (26)$$

$$\frac{\partial^2 C}{\partial T\,\partial T_1}=\frac{\left(h+\theta C_d\right)}{24}\Big[24aT_1+12T_1^2\left(a\theta-2b\right)+4\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^3+$$

$$5\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^4\Big]-\frac{aC_sT_1}{T^2} \quad (27)$$

For extremum putting $\frac{\partial C}{\partial T_1}=0$ and $\frac{\partial C}{\partial T}=0$, we get

$$24a\left(h+\theta C_d\right)T_1+12\left(h+\theta C_d\right)\left(a\theta-2b\right)T_1^2+4\left(h+\theta C_d\right)\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^3+$$

$$5\left(h+\theta C_d\right)\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^4-24aC_s\left(T-T_1\right)=0 \quad (28)$$

$$24C_0+\left(h+\theta C_d\right)\Big[24aT_1+12T_1^2\left(a\theta-2b\right)+4\left\{-4b+\left(-4a+7b\right)\theta-a\theta^2\right\}T_1^3+$$

$$5\left\{2\theta\left(a\theta-b\right)\left(\theta-1\right)+\theta^3\right\}T_1^4\Big]-12aC_s\left(T^2-T_1^2\right)=0 \quad (29)$$

Solving (28) and (29) simultaneously for T_1 and T for finding optimal values of T_1 and T respectively, which minimizes C.

5. Numerical Examples

5.1. Example 1 (Model I)

Let us consider a= 4500, b = 0.5, C_0 = 100, h =10, C_d = 100, α = 0.04, β = 0.01, C_d = 100 in suitable units. Replace with these in (11), we obtain optimal value of T = 0.0523957, which minimizes total cost C, and corresponding values of SU = \$1908.55, HC = \$1181.36, DC = \$120.028, C = \$ 39015.6. The pictorial representation of the model I is as follows:

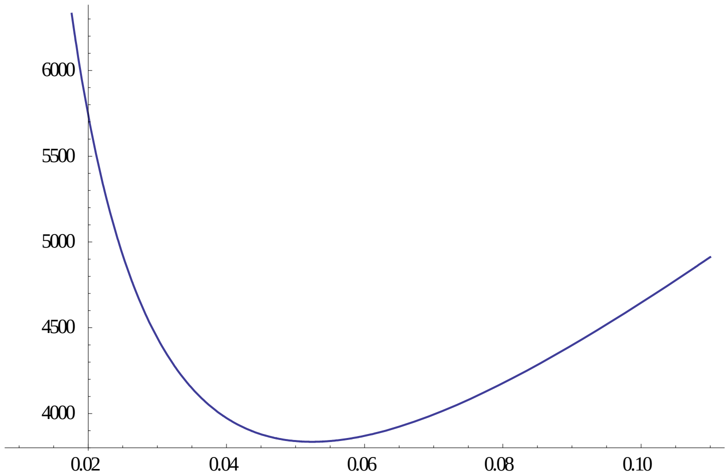

Fig. 3. Pictorial representation C vs. T

5.2. Example 2 (Model II)

Consider a = 500, b = 0.5, C_0= 100, C_h =10, C_s =10, θ= 0.01 - 0.10; C_d =10 in appropriate units. On substitution these values in Eq. (24) & (28), solving simultaneously, we get optimal T = 1.22581, and T_1 = 1.10484, corresponding SU = \$ 81.5787, HC = \$ 250.590, DC = \$ 25.059, SC = \$ 29.8450, C = \$ 353.128. Figure of model II is given below:

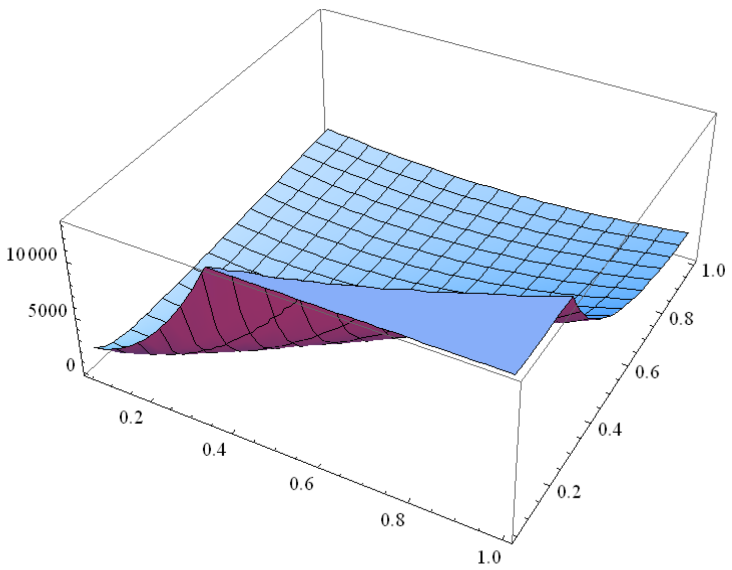

Fig. 4. Pictorial representation between C vs. T_1 and T

6. Sensitivity Analysis

6.1. Model I

Using the identical information as those in example 1, we learn sensitivity investigation on optimal solution with respect to parameters α, β and C_0. The computational results are shown in Tables 1 (a, b and c).

Table 1 (a): Variation of T, Q, SU, HC, DC and C with α

α	T	Q	SU	HC	DC	C	$\frac{d^2C}{dT^2}$
0.05	0.0477652	215.199	2093.57	1077.280	109.265	46627.0	$1.82023\text{x}10^6 > 0$
0.06	0.0441736	199.044	2263.80	996.533	100.939	54223.5	$1.82023\text{x}10^6 > 0$
0.07	0.0412831	186.042	2422.30	931.548	94.2522	61808.3	$1.82023\text{x}10^6 > 0$
0.08	0.0388919	175.285	2571.23	877.785	88.7297	69383.6	$1.82023\text{x}10^6 > 0$
0.09	0.0368711	166.195	2712.15	832.347	84.0692	76950.8	$1.82023\text{x}10^6 > 0$

(b): Variation of T, Q, SU, HC, DC and C with β

β	T	Q	SU	HC	DC	C	$\frac{d^2C}{dT^2}$
0.02	0.0477907	215.263	2092.46	1077.34	109.303	46624.3	$1.81629\text{x}10^6 > 0$
0.03	0.0442170	199.152	2261.57	996.633	101.003	54218.2	$2.29473\text{x}10^6 > 0$
0.04	0.0413397	186.182	2418.98	931.672	94.3348	61800.3	$2.80948\text{x}10^6 > 0$
0.05	0.0389587	175.450	2566.82	877.927	88.8262	69372.9	$3.35818\text{x}10^6 > 0$
0.06	0.0369460	166.380	2706.65	832.503	84.1764	76937.5	$3.93889\text{x}10^6 > 0$

(c): Variation of *T* , *Q, SU* , *HC, DC* and *C* with C_0

C_0	T	Q	SU	HC	DC	C	$\frac{d^2C}{dT^2}$
110	0.054979	247.677	2000.76	1239.74	126.056	39106.0	$1.31051 \times 10^6 > 0$
120	0.0574497	258.820	2088.78	1295.58	131.832	39192.3	$1.25244 \times 10^6 > 0$
130	0.0598215	269.518	2173.13	1349.19	137.385	39274.8	$1.20120 \times 10^6 > 0$
140	0.0621057	279.822	2254.22	1400.84	142.741	39354.1	$1.15555 \times 10^6 > 0$
150	0.0643115	289.773	2332.40	1450.72	147.921	39430.4	$1.11455 \times 10^6 > 0$

From tables 1, we summarize the following results: (i) Increase of parameters α and β results, decrease in *T, Q, HC, DC*, and increase in *SU* and *C* and (ii) Increase of parameter C_0 results decrease in *T, Q* , *HC, DC, SU* and *C*.

6.2. Model II

Using the equivalent data as those in example 2, we study sensitivity examination on optimal solution with respect to each parameter in proper units. Computational results are shown in tables 2 (a - g)

Table 2 (a): Variation of *T* , T_1 , *Q* , *SU* , *HC, DC, SC* and *C* with θ

θ	T	T_1	Q	SU	HC	DC	SC	C
0.01	1.22581	1.10484	616.663	81.5787	250.590	25.0590	29.8450	387.072
0.02	1.21899	1.08921	617.176	82.0357	246.702	49.3403	34.5426	412.568
0.03	1.21367	1.07521	618.221	82.3947	246.125	72.9376	39.4901	437.947
0.04	1.21367	1.06253	619.681	82.6754	239.805	95.9219	44.6755	463.078
0.05	1.20638	1.05093	621.459	82.8926	236.699	118.350	50.0769	488.018
0.06	1.20401	1.04023	623.500	83.0558	233.770	140.262	55.6970	512.785
0.07	1.20228	1.03029	625.739	83.1753	230.997	161.698	61.5093	537.380
0.08	1.20110	1.02101	628.146	83.2570	228.361	182.789	67.5056	561.912
0.09	1.20038	1.01228	630.683	83.3070	225.836	203.253	73.6881	586.084
0.10	1.20004	1.00405	633.322	83.3306	223.356	223.423	80.0225	610.132

Sensitivity Analysis

Let $r = \frac{\partial^2 C}{\partial T^2}$, t $= \frac{\partial^2 C}{\partial t_1^2}$ and $s = \frac{\partial^2 C}{\partial T \partial t_1}$.

Calculation of *'r'*, *'s'*, *'t'*				
θ	r	s	t	$rt - s^2$
0.01	2765.39 > 0	-3071.53 < 0	4519.44 > 0	3063717.639 > 0
0.02	2668.78 > 0	-3016.15 < 0	4578.62 > 0	3122168.661 > 0
0.03	2570.73 > 0	-2957.43 < 0	4631.89 > 0	3160946.375 > 0
0.04	2472.14 > 0	-2896.22 < 0	4680.12 > 0	3181821.518 > 0
0.05	2373.80 > 0	-2833.29 < 0	4724.11 > 0	3186560.094 > 0
0.06	2276.07 > 0	-2769.00 < 0	4764.32 > 0	3176564.822 > 0
0.07	2179.45 > 0	-2703.89 < 0	4801.28 > 0	3153128.564 > 0
0.08	2084.18 > 0	-2638.21 < 0	4835.29 > 0	3117462.708 > 0
0.09	1990.37 > 0	-2572.18 < 0	4866.67 > 0	3070364.016 > 0
0.10	1898.35 > 0	-2506.10 < 0	4895.70 > 0	3013214.885 > 0

Table 2(b): Variation of T , T_1 , Q , SU , HC, DC, SC and C with 'a'

a	T	T_1	Q	SU	HC	DC	SC	C
600	1.19991	1.08143	724.321	83.3396	294.310	29.4310	35.0964	404.447
700	1.18080	1.06416	831.546	84.6883	337.853	33.7853	40.3261	456.675
800	1.16610	1.05088	938.472	85.7559	381.277	38.1277	45.5386	507.560
900	1.15443	1.04035	1045.18	86.6228	424.622	42.4622	50.7299	558.083
1000	1.14495	1.03178	1151.75	87.3401	467.893	46.7893	55.9302	608.348

Calculation of *'r'*, *'s'*, *'t'*				
a	r	s	t	$rt - s^2$
600	3354.44 > 0	-3795.74 < 0	5541.32 > 0	4180383.313 > 0
700	3944.50 > 0	-4526.13 < 0	6570.27 > 0	5430577.238 > 0
800	4535.24 > 0	-5260.86 < 0	7604.20 > 0	6810224.068 > 0
900	5126.58 > 0	-5998.85 < 0	8641.78 > 0	8316575.190 > 0
1000	5717.99 > 0	-6739.00 < 0	9681.99 > 0	9947401.000 > 0

Table 2 (c): Variation of T , T_1 , Q , SU , HC, DC, SC and C with **'***b***'**

b	T	T_1	Q	SU	HC	DC	SC	C
0.6	1.22584	1.10491	616.595	81.5767	250.560	25.0560	29.8246	352.502
0.7	1.22588	1.10498	616.532	81.5741	250.527	25.0527	29.8088	351.878
0.8	1.22591	1.10505	616.464	81.5721	250.496	25.0496	29.7884	351.252
0.9	1.22594	1.10512	616.396	81.5701	250.465	25.0465	29.7679	350.626
1.0	1.22598	1.10520	616.333	81.5674	250.437	25.0437	29.7472	350.000

Calculation of *'r'*, *'s'*, *'t'*				
b	r	s	t	$rt - s^2$
0.6	2765.81 > 0	-3071.77 < 0	4518.91 > 0	3062675.534 > 0
0.7	2766.17 > 0	-3071.96 < 0	4518.35 > 0	3061585.978 > 0
0.8	2766.60 > 0	-3072.21 < 0	4517.83 > 0	3060554.194 > 0
0.9	2767.03 > 0	-3072.45 < 0	4517.31 > 0	3058231.360 > 0
1.0	2767.44 > 0	-3072.67 < 0	4516.75 > 0	3058533.691 > 0

Table 2 (d): Variation of T, T_1, Q, SU, HC, DC, SC and C with C_0

C_0	T	T_1	Q	SU	HC	DC	SC	C
110	1.24090	1.11845	624.314	88.6453	253.711	25.3711	30.2079	362.297
120	1.25567	1.13179	631.804	95.5665	256.775	25.6775	30.5539	371.225
130	1.27016	1.14487	639.155	102.349	259.779	25.9779	30.8969	379.899
140	1.28437	1.15770	646.364	109.003	262.727	26.2727	31.2318	388.358
150	1.29833	1.17030	653.449	107.831	265.622	26.5622	31.5630	396.606

Calculation of *'r'*, *'s'*, *'t'*				
C_0	r	s	t	$rt - s^2$
110	2747.68 > 0	-3019.50 < 0	4464.23 > 0	3148895.236 > 0
120	2730.50 > 0	-2969.68 < 0	4411.47 > 0	3226519.533 > 0
130	2713.65 > 0	-2921.74 < 0	4630.90 > 0	4030077.157 > 0
140	2697.20 > 0	-2875.63 < 0	4312.42 > 0	3615935.655 > 0
150	2681.08 > 0	-2831.17 < 0	4265.81 > 0	3421454.306 > 0

Table 2 (e): Variation of T, T_1, Q, SU, HC, DC, SC and C with C_h

C_h	T	T_1	Q	SU	HC	DC	SC	C
2	1.20014	0.992422	603.019	83.3236	412.619	20.6310	89.8786	583.746
3	1.21305	0.926570	609.053	82.4368	533.483	17.7828	169.141	785.149
4	1.23507	0.876483	619.769	80.9671	624.890	15.6223	260.278	967.254
5	1.25952	0.834624	631.763	79.3953	694.313	13.8863	358.344	1133.75
6	1.28420	0.798091	643.914	77.8695	746.988	12.4498	460.018	1286.92

Calculation of *'r'*, *'s'*, *'t'*				
C_h	r	s	t	$rt - s^2$
2	1762.86 > 0	-2406.50 < 0	5028.93 > 0	3074057.290 > 0
3	911.953 > 0	-1715.99 < 0	5384.78 > 0	1966044.595 > 0
4	241.660 > 0	-1080.05 < 0	5691.58 > 0	208919.2203 > 0
5	-283.248 < 0	-506.104 < 0	5976.64 > 0	-1949012.57 < 0*
6	-697.072 < 0	10.8494 > 0	6250.18 > 0	-4356943.182 < 0*

*represents the non- optimal points

Table 2 (f): Variation of T , T_1 , Q , SU , HC, DC, SC and C with C_d

C_d	T	T_1	Q	SU	HC	DC	SC	C
20	1.21880	1.08872	613.035	82.0479	244.695	48.939	34.71	378.293
30	1.21335	1.07428	610.202	82.4165	239.286	71.774	39.85	402.826
40	1.20915	1.06119	608.005	82.7027	234.274	93.710	45.26	426.818
50	1.20596	1.04920	606.323	82.9215	229.591	114.80	50.94	450.337
60	1.20360	1.03813	605.064	83.0841	225.191	135.12	56.87	473.434

Calculation of ‘r’, ‘s’, ‘t’				
C_d	r	s	t	$rt-s^2$
20	2665.47 > 0	-3014.19 < 0	4586.03 > 0	3138584.028 > 0
30	2563.78 > 0	-2953.17 < 0	4647.42 > 0	3193749.399 > 0
40	2461.20 > 0	-2889.32 < 0	4704.51 > 0	3230569.950 > 0
50	2358.48 > 0	-2823.32 < 0	4758.03 > 0	3250582.772 > 0
60	2256.18 > 0	-2755.74 < 0	4808.52 > 0	3254783.706 > 0

Table 2 (g): Variation of T , T_1 , Q , SU , HC, DC, SC and C with C_s

C_s	T	T_1	Q	SU	HC	DC	SC	C
12	1.21487	1.11330	611.259	82.3133	256.754	25.6754	25.4755	355.211
14	1.20697	1.11943	607.357	82.8521	261.303	26.1303	22.2221	356.713
16	1.20099	1.12408	604.403	83.2646	264.802	26.4802	19.7009	357.851
18	1.19631	1.12773	602.092	83.5904	267.576	26.7576	17.6915	358.739
20	1.19254	1.13066	600.230	83.8546	269.826	26.9826	16.0545	359.452

Calculation of ‘r’, ‘s’, ‘t’				
C_s	r	s	t	$rt-s^2$
12	3579.25 > 0	-3916.45 < 0	5383.11 > 0	3928915.865 > 0
14	4405.64 > 0	-4766.26 < 0	6246.75 > 0	4803697.282 > 0
16	5240.45 > 0	-5619.34 < 0	7110.41 > 0	5684766.049 > 0
18	6081.11 > 0	-6474.63 < 0	7974.06 > 0	6570302.370 > 0
20	6926.00 > 0	-7331.54 < 0	8837.76 > 0	7458846.988 > 0

The sensitivity analysis reveals that: (i) a higher value of θ causes lower value of T , T_1 , HC and higher value of Q , SU , DC, SC and C. (ii).a higher value of parameter ‘a’ causes lower value of T , T_1 and higher values of Q , SU, HC , DC , SC and C. (iii) a higher value of parameter ‘b’ causes higher values of T , T_1 and lower values of Q , SU , HC, DC,SC and C. (iv) a higher value of C_0 causes higher values of T , T_1 ,Q , SU, HC , DC , SC , and C.(v) a higher value of C_h results higher values of T , Q , HC , SC, C and lower values of T_1 , SU and DC. (vi). a higher value of C_d results decrease in T , T_1 , Q , SU , HC and increase in DC , SC, and C.

Research Gap: Previous researchers have discussed models in which demand and deterioration both are constant while in this model both are variable. We have also modeled for shortages.

7. Conclusion and Future Research

In this paper, a purchasing EOQ model for spoiling commodities by means of non- increasing demand under stock- dependent holding cost with and without shortage. Very few similar models have been studied in the literature. The existence of multiple items that have different deteriorating commodities is formulated. To avoid complexity in modelling, truncated Taylor' series approximation for exponential terms is used. In this study an approximate inventory models have been developed considering different deterioration rates. In the sensitivity analysis conducted in this paper, it is shown that (i) greater value of α, β results less value of Q and greater value of C (ii) greater values of θ , a , C_o and C_h results greater values in Q and C (iii) greater values of C_d results diminish value of Q and higher value of C and (iv) greater values of 'b' results slight decrease in Q and C.

The model presented in this study may be extended by including freight charges. We could also generalize it by considering advertisement costs and others.

References

[1] Yang, P.C. and Wee, H.M.; Economic ordering policy of deteriorated item for vendor and buyer: an integrated approach. *Production Planning and control*, 11(5)(200): 474-480.

[2] Yan, C. Banerjee. A. and Yang, L. An integrated production-distribution model for a deteriorating inventory item. *International Journal of Production Economics*, 133(2011): 228-232

[3] Ghiami, Y. and Williams, T. A two-echelon production-inventory model for deteriorating items with multiple buyers. *International Journal of Production Economics*, 159(2015): 233-240.

[4] Lee, S. and Kim, D. An optimal policy for a single-vendor, single buyer integrated production distribution model with both deteriorating defective item. *International Journal of Production Economics,* 147(2014): 163-170.

[5] Ghare, P.M. and Schrader, G.H. A model for an exponentially decaying inventory. *Journal of Industrial Engineering*, 14(5)(1963): 238-243.

[6] Dye, C.Y. The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. *Omega*, 41(2013): 872-880.

[7] Abad, P.L. Optimal pricing and lot-sizeing under condition of pershability and partial backordering. *Management Science*, 42(8)(1996): 1093-1104.

[8] Wee, H.M. A replenishment policy for item with a price-dependent demand and varying rate of deterioration. *Production Planning and Control*, 8(6)(1997): 494-499.

[9] Tsao, Y.C. and Sheen, G.J. Dynamic pricing promotion and replenishment policies for a deteriorating item under permissible delay in payments. *Computers and Operations Research*, 35(11)(2008): 3562-3580.

[10] Wang, X.J. and Li, D. A dynamic product quality evaluation based pricing model for perishable food supply chain. *Omega*, 40(6)(2012): 906-917.

[11] Das, B.C. Das, B. and Mondal,, S.K. An integrated production inventory model under interactive fuzzy credit period for deteriorating item with several markets. *Applied Soft Computing*, 28(2015): 453-465

[12] Taleizadeh, A.A., Noori-daryan, M. and Cardenas Barron, L.E. Joint optimization of price, replenishment cycle and production rate in vendor managed inventory system deteriorating items. *International Journal Production Economics*, 159(28)(2015): 285-295.

[13] Dave, U. and Patel, I.K. Policy inventory model for deteriorating item with time proportional demand. *Journal of Operational Research Society*, 32(1981): 137-142.

[14] Xu, H. and Wang, H. (1991). An Economic ordering policy model for deteriorating item with time proportional demand. *European Journal of Operational Research*, 7(1991): 21-24

[15] Sarkar, B. Sana, S.S. and Chaudhuri, K.S. An imperfect production processes for time varying demand with inflation and time value of money-An EMQ model. *Expert System with Application*, 38(2011): 13543-13548.

[16] Ghosh, S.K. and Chaudhuri, K.S. An EOQ model with a quadratic demand, time proportion deterioration and shortage in all cycle. *International Journal of System Science*, 37(10)(2006): 663-672.

[17] Ghosh, S.K. and Sarkar, T. and Chaudhuri, K.S. An optimal inventory replenishment policy for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortage in all cycle. *Applied Mathematics and Computation*, 218(2012): 9147-9155.

[18] Chen, J.M. An inventory model for deteriorating item with time-proportional demand and shortage under inflation and time discounting. *International Journal of Production Economics*, 55(1)(1998): 21-30.

[19] Lee, C.C. and Hsu, S.I. A two warehouse production model for deteriorating inventory item with time-dependent demand. *European Journal of Operational Research*, 194(3)(2009): 700-710.

[20] Hsich, T.P. and Dye, C.Y. A production inventory model in corporating the effect of preservation technology investment when demand is fluctuating with time. *Journal of Computational and Applied Mathematics*, 239(2013): 25-36.

[21] Wee, H.M. and Wang, W.T. A variable production scheduling policy for deteriorating item with time-varying demand. *Computers and Operations Research*, 26(3)(1999): 237-254.

[22] Shah, N.H., Shah, D.B. and Patel, D.G. Optimal credit period and purchase quantity for credit dependent trended demand. *Journal of Operational Research Society*, 51(1)(2015): 101-107.

[23] Shukla, H.S. Shukla, V. and Yadav, S.K.(2013). EOQ model for deteriorating item with exponential demand rate and shortage. *Uncertain Supply chain Management*, 1(2013): 67-76.

[24] Mahata, G.C. EOQ model for item with exponential distribution deterioration and linear trend demand under permissible delay in payment. *International Journal of Soft Computing*, 6(3)(2011): 46-53.

[25] Tripathi, R.P., Singh, D and Aneja, S. Inventory control model using discount cash flow approach under multiple supplier's trade credit and stock dependent demand for deteriorating items. *International Journal of Inventory Research*, 5(3)(2019): 210-223.

[26] Tripathi, R.P., Singh, D and Aneja, S. Inventory models for stock-dependent demand and time varying holding cost under different trade credits. *Yugoslaw Journal of Operations Research*, 28(1)(2018): 139-151.

[27] Tripathi, R.P., Pareek, S. And Kaur, M. Inventory model with exponential two-dependent demand rate, variable deterioration, Shortages and Production Cost. *International Journal of Applied and Computational Mathematics*, 3(2)(2017): 1407-1419.

[28] Tripathi, R.P. Economic order quantity models for price-dependent demand and different holding cost function. *Jordan Journal of Mathematics and Statistics*, 12(1)(2019): 15-33.

[29] Nobil, A.H., Sedigh, H.A. An economic production quantity inventory model with a defective production system and uncertain upline. *International Journal of Inventory Research*, 4(2-3)(2017): 123-147.

[30] Alfares, H.K., Ghaithan, A.M. (2019). EOQ and EPQ production – inventory models with variable holding cost: State -of -the - Art Review. *Arabian journal for science and Engineering*, 44(2019): 1737-1755.

[31] Tripathi, R.P. Establishment of EOQ (Economic Order Quantity) models for spoilage products and power demand under permissible delay in payment. *International Journal of Applied and Computational Mathematics, 2018.*

[32] Tripathi, R.P., Singh, D., Singh, D. (2019). Inventory model with quantity discount, pricing and partial- backlogging. *International Journal of Operational Research*, 35(2)(2019): 208-223.

[33] Taleizadesh, A.A., Bayatloo, F. (2017). Retailer's reaction in respect to price increase: pricing and order decisions. *International Journal of Supply Chain and Inventory Management*, 2(2)(2017): 178-195.