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Article

Precision Estimation of Assay Data in Mine Exploration Using Robust Regression

Joseph Acquah [1], Kofi Agyarko [2,*] and Peter Ofori-Amanfo [3]

[1]*University of Mines and Technology, Department of Mathematical Sciences, Tarkwa, Ghana.*

[2]*University of Mines and Technology, Department of Mathematical Sciences, Tarkwa, Ghana*

[3]*Prestea Mine, Prestea, Ghana*

*Author to whom correspondence should be addressed; E-Mail: kofiagyarko88@gmail.com

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Abstract: The paper examined the use of robust regression techniques in solving problems associated with outliers or extreme observations. Thus, the study seeks for a parameter estimation method which is robust in nature such that a small change in the data set have no effect on the value of the estimation. The robust regression methods considered to determine an acceptable regression model to use are the M-estimate, the MM-estimate, the S-estimate, Ordinary Least Squares (OLS), and the Least Absolute Value (LAV) method. The algorithms of these methods are presented and applied to an assay data in mine exploration to determine the precision estimates in assessing the repeatability of the data. The results show that the use of robust regression techniques in estimating precision of assay data in mine exploration is feasible and reliable.

Keywords: Precision Estimate, Assay Data, Robust Regression, M-estimation, S-estimation, MM-estimation, and Least Absolute Value (LAV) method

Mathematics Subject Classification: Applied Statistics

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1. Introduction

Precision can be defined as how close an estimate is to other samples' estimate. It can also be defined as the reproducibility of same result from a laboratory experiment (McAlinden *et al.,* 2015). In other words, it is the degree of detail with which an experimental value is accurately reported. Precision in mine exploration is thus determined by submitting duplicate samples from the same ore source to assess its repeatability. The duplicate samples are usually either from the primary sampling stage (field duplicate) or secondary sampling stage (subsampling at the laboratory duplicates). The original and the duplicate sample are given different sample identification numbers. Field duplicates are submitted with the original and the lab duplicates are submitted after the original sample results are reported. Precision needs to be determined at its best-known accuracy for which measurement can be recorded. But at times, it is misleading when higher precision is achieved beyond a known accuracy of a given measurement. Also, the results of a given sample can be said to be of low precision when multiple analyses of the same sample or duplicate show a wide variation in the results. Analytical precision in mine exploration is acceptable if results of the replicate analysis of assay data conducted in the laboratory satisfy a statistical test (ISO Guide 33). The result of the assay data obtained from different statistical test conducted can be used for prediction of the ore concentration in mine exploration.

Assay data is the result or information obtained after a field sample has been analysed with advanced chemical in the laboratory. Assaying is the process of analysing a physical sample to determine its composition (Dominy *et al.,* 2018). In mine exploration, the term assay usually refers to the chemical analysis of a mineral or ore sample to ascertain its content of precious metals or minerals, such as gold, copper, or uranium etc. These samples are obtained from drill holes made in a mineral or ore body, or property that is being evaluated for its mineral content by a resource exploration or production company.

Precision error of assayed samples is estimated and monitored by using matching of matching pairs of data. The first sample is usually referred to as original sample, and the second is called duplicate. The duplicate sample is merely another sample collected from the same place or rock type, following the same rules as used for collecting the initial (original) sample. In practice, the duplicate sample can be a second blast hole sample collected from the same blast hole cone or another half of the drill core. It can also be a duplicate sample collected at a particular stage of the sampling procedure. Usually, it is a good practice to take duplicate samples at each stage of the crushed material.

1.1. Errors in Assayed Samples

The fundamental cause of the errors of samples of rocks and minerals collected by geologists for evaluation of mining projects is the heterogeneity nature of the sampled materials (Gy, 1982; Francois-Bongarcon, 1993; Pitard, 1993). The more heterogeneous the sampled material, the more difficult it is to obtain a representative sample and infer characteristics of the geological object from samples. In recent studies, sampling theory explaining sampling error types and their likely causes, describes the practical approaches used in the mining industry for estimating sampling errors and monitoring them as an acceptably low level of accuracy. It is based on numerous case studies by different author's (Abzalov and Both, 1997; Abzalov, 2007, 2008; Abzalov and Humphreys, 2002; Abzalov and Mazzoni, 2004; Abzalov and Pickers, 2005; Abzalov *et al.*, 2007; Abzalov and Bower, 2009). Also, reviews of the recently published Quality Assurance and Quality Control (QAQC) procedures used in the mining industry (Taylor, 1987; Vallee *et al.,* 1992; Leaver *et al.,* 1997; Long, 1998; Sketchley, 1998), revealed that the control of analytical data quality has gone down. Thus, the difference between the original value and the duplicate value for any sample assay prediction is very important in ore mineralogy. These errors can therefore be classified into the following genres: Fundamental Sampling error, Analytical and Instrumental errors, Delimitation, Extraction and Preparation errors, and Weighing errors etc. These errors can be generated at any stage of the sampling error.

2. Materials and Methods

2.1. Precision Estimation Techniques

Various statistics and geo-statistics techniques have been explored in estimating acceptable precision errors for paired or duplicated samples in mine mineralogy. Some of the methods in line with this study that are widely used in the mining industry are: Absolute Mean Percentage Difference (AMPD), Half Absolute Relative Difference (HARD), and the Reduce Major Axis (RMA), (Stanley and Lawie, 2007a; Sinclair and Bentzen, 1998). The AMPD and the HARD both makes use of Coefficient of Variation (CV). The functional relation of AMPD and HARD according to Stanley and Lawie (2007a) is given by: $\sqrt{2}xCV(\%)$ and $\frac{\sqrt{2}}{2}$x $CV(\%)$ respectively, where the coefficient of variation CV =100 x $\frac{Standard\ Deviation}{Mean}$. Stanley and Lawie (2007a) noted in their study that using such statistics, are directly proportional to using the coefficient of variation, since both methods offers no more information than the CV itself. The RMA on the other hand, is a regression technique for modelling error precisions in paired data (Sinclair and Bentzen, 1998; Davis, 2002; Sinclair and Blackwell, 2002). This technique minimizes the product of the deviations in both directions and both sample data. The RMA is thus a useful tool for testing duplicated data in the presence of outliers. The relative precision error (P_{RMA} *(%)*) of the RMA model can be defined as:

$$P_{RMA}(\%) = 100\text{x}\sqrt{\frac{\frac{S_{RMA}^2}{2}}{C}} \quad (1)$$

where *C* is the mean grade of the paired data.

Basically, most of these techniques are linear regression approaches. Linear regression is thus a method that model the relationship between a response variable y, and one or more explanatory variables. In linear regression, data are modelled using linear predictor functions, and unknown model parameters are estimated from the data (Cohen, *et al.,* 2003). A linear regression model that involves j independent variables can be expressed as

$$y_i = \sum_{j=0}^{m} \beta_j x_i + \varepsilon_i, \quad (2)$$

where y_i is the response variable on the i^{th} observations, $\beta_0, \beta_1, ..., \beta_j$ are parameters x_i is the values of the independent variables and ε_i is the error term which is assumed to be normally distributed. The regression method which is commonly used in addition to the above-mentioned techniques is the ordinary least square (OLS) method. However, the OLS method is often affected by outliers. Hence, the need for a more robust regression technique as employed in this study. Other candidate models have also been explored. The work of Eremeev *et al*. (1982), estimated precision error as a ratio of the standard deviation (S) of the duplicated data to the mean of the data pairs in a given grade class. A conventional formula that applies relative variogram to the square root of the duplicated samples to estimate precision error was also proposed by Goovaert (1997).

2.2. The Robust Regression Methods

Robust regression analysis provides an alternative to the least squares regression model when fundamental assumptions are not met by the nature of the data. Thus, the distribution of residuals is said not to be normal or there are some outliers that affect a defined fitted model. In such situations it is sometimes good to transform the data to conform to these fundamental assumptions. Most often, a transformation will not eliminate the leverage of influential outliers that affects the prediction or distort the significance of parameter estimates. Under these circumstances, a regression technique that is resistant to the influence of outliers may be the only reasonable choice.

A technique that estimate the parameter β reliably with an exact identification of the outliers or taking into consideration all outliers is termed as Robust regression. In robust regression, the properties of efficiency and high leverage points are used to define the kind of robust technique one seeks theoretically. Some of the robust techniques employed in this study are: the M-estimate, the MM-estimate, the S-estimate and the Least Absolute Value (LAV) method.

2.3. The M-Estimator

The most common general method of robust regression is the M-estimation, which was introduced by Huber and Rigley (1964) that is as efficient as OLS. In the M-estimation, instead of minimizing the sum of squared errors as the objective function as in the OLS, the M-estimate rather minimizes a function η of the errors. The objective function of the M-estimate is given by

$$\min \sum_{i=1}^{n} \eta\left(\frac{e_i}{\sigma}\right) = \min \sum_{i=1}^{n} \eta\left(\frac{y_i - x_i\hat{\beta}}{\sigma}\right) \tag{3}$$

where σ is an estimate of scale often formed from linear combination of the residuals. The function η gives the contribution of each residual to the objective function. A reasonable η should have the following properties: $\eta(e) \geq 0, \eta(o) = 0, \eta(-e) \geq \eta(e)$ and $\eta(e_i) \geq \eta(e_i^{'})$ for $|e_i| \geq |e_i^{'}|$ (Samkar and Alpu, 2010)

2.4. The S-Estimator

The S-estimation is a high breakdown value method introduced by Rousseeuw and Yohai (1984) that minimizes the dispersion of the residuals. The objectives function is $\min \sigma(e_1(\beta),...,e_n(\beta))$ where $e_i(\beta)$ is the i^{th} residuals for candidate β. This objective function is given by the solution

$$\frac{1}{n-p} \sum_{i=1}^{n} \eta\left(\frac{y_i - \hat{y}_i}{\sigma}\right) = K_p \tag{4}$$

where K_p is a constant at the defined parameter point p, $E_\theta[\eta]$ with θ defined as the standard normal (Rousseeuw and Yohai, 1984).

2.5. The MM-Estimation

MM-estimation is a special type of M-estimation developed by Yohai (1987). The MM-estimator have three stage procedures (Susanti and Pratiwi 2014)

(i) The first stage is calculating an S-estimate with influence function

$$\eta(x) = 3\left(\frac{x}{c}\right)^2 - 3\left(\frac{x}{c}\right)^4 + \left(\frac{x}{c}\right)^6 \text{ if } |x| \leq c \text{ otherwise } \eta(x) = 1 \tag{5}$$

(ii) The second stage calculate the MM parameters that provide the minimum value of $\sum_{i=1}^{n} \eta\left(\frac{y_i - x_i^{'}\beta_{MM}}{\hat{\sigma}_0}\right)$ where $\eta(x)$ is the influence function used in the first stage and $\hat{\sigma}_0$ is the estimate of scale standard deviation of the residuals.

(iii) The final stage calculates the MM estimate of scale function as the solution to the equation

$$\frac{1}{n}\sum_{i=1}^{n}\eta\left(\frac{y_i - x_i'\hat{\beta}}{\sigma}\right) = 0.5$$

2.6. Least Absolute Values Regression

Least absolute values (LAV) regression is very resistant to observations with unusual y values. Estimates are found by minimizing the sum of the absolute values of the residuals

$$min\sum_{i=1}^{n}\lceil e_i\rceil = min\sum_{i=1}^{n}\lceil y_i - \sum x_{ij}\beta_j\rceil \quad (6)$$

The LAV is a more general quantile regression with the objective function as a minimization of the function above (Koenker and Bassett, 1978; Koenker, 2005).

3. Results and Discussion

The methods outline in section 2 were tested using data obtained from a Hypothetical mine in the Tarkwa-Prestea Mining belt of the Western Region of Ghana. The data obtained from the mine is shown in Appendix A. The data is made up of two different sets. The first set is the assumed original data whilst the second is the duplicated sample. Both the original and the duplicated samples were obtained from fire assay experiment conducted at the lab by the mine. The data was first assessed to check for outliers. Two of the most explored techniques in mine exploration were first assessed to determine the feasibility of the data. Figure 1 shows the scattergram and the fitted RMA plot of the data. Figure 2 also shows the fitted HARD plot of the original and the duplicated samples. Figure 1 shows the presence of outliers in the data along the fitted RMA plots, whilst Figure 2 shows some extreme outliers above the fitted trendline. Table 1 on the other hand gives the estimated precision error, the standard error and the correlation coefficients for the three techniques are applied to analyse the data. The estimated precision error for both RMA and HARD were determined as 22.9 % and 6.8 % respectively. Thus, the percentage precision estimates for RMA is 77.1% and that of HARD is also 93.2%. According to the industry standards (ISO 33), if the precision error is less than 15% the proposed method can be adapted and used to assess the repeatability of different assay data. From the estimated precision estimates, it can be inferred that the HARD has a better precision estimates than the RMA with over 90% certainty of a good repeatability. The linear regression method used on the other hand has an error precision estimate of 16.7%, an indication that is also better conditioned than the RMA.

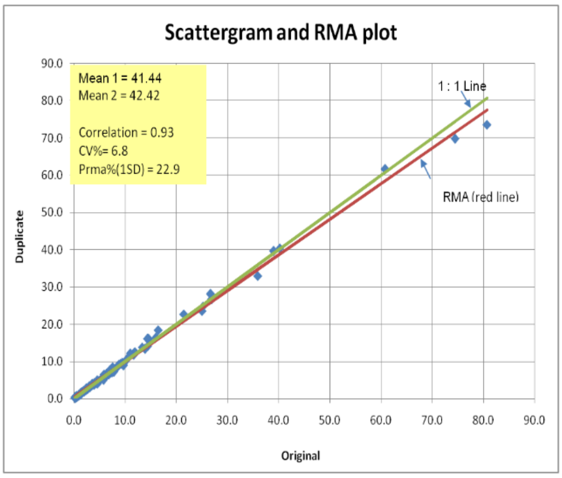

Figure 1. Scattergram and RMA Plot

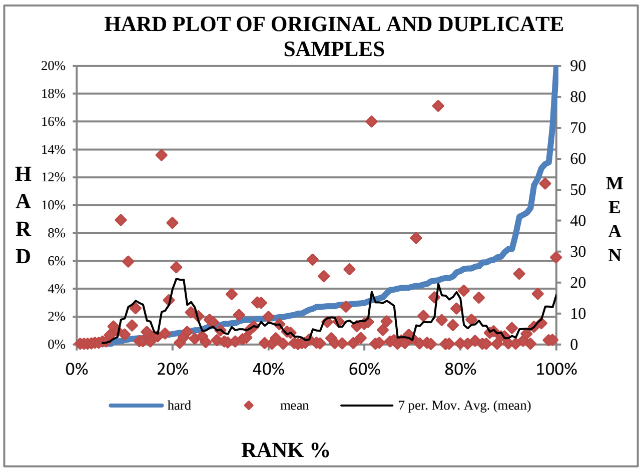

Figure 2. Fitted HARD Plot of Data

Table 1. Parameter estimates for RMA and HARD

MODEL	PRECISION ERROR	CORRELATION COEFFICIENT	STANDARD ERROR
RMA	22.9 %	0.93	0.299
HARD	6.8 %	0.91	0.068
REG.	16.3 %	0.99	0.163

3.1. Application of Robust Regression Techniques

The parameter estimates of the desired model was determined based on these selected robust regression methods: Ordinary Least Squares (OLS), M-estimation with Huber Weight (M-HUBER), M-Estimation with Bisquare Weight (M-BISQUARE), MM-estimation (MM), S-estimation (S) and the Least Absolute Value (LAV). The selected methods were used to analyse a mine assay data to assess the precision estimates of the data as well as the efficiency of the methods. The simulated results are presented as shown in Table 2. Table 2 is made up of the explored methods, the parameters (β_0, β_1) with their respective estimated values as well as their standard errors, and the average estimated precision errors. It can also be observed from Table 2 that the estimated $\hat{\beta}_1$ value for the OLS method was observed to be highest with a value of 1.03621, followed by LAV (0.9928), MM (0.9871), M-HUBBER (0.9929), S (0.9867) and M-BISQUARE (0.9865) in that order. The coefficient of the explanatory variable with a positive value 1.03621 for OLS could have effects on the dependent variable. Again, it can be observed from the Table 2 that the OLS have the highest standard error with an average value of 0.988. This shows that there are outliers in the data and have trickling effect on the OLS method explored.

However, the other robust methods explored (LAV, MM, S, M-HUBER, and M-BISQUARE) have coefficient of the explanatory variable very close to that of the OLS but with a minimal standard error estimates in all the methods used compared with that of the OLS method. The standard error for the coefficient ($\hat{\beta}_1$) of the M-estimation with Bisquare weight was observed to have the least standard error estimate with an estimated value of 0.001, compared to all the methods explored.

Table 2. Parameter estimates and their standard errors

Method	Parameter	β_0	β_1	Av. Precision Error(%)
OLS	Estimate	-0.2987	1.0362	
	Stand. Err	0.1854	0.0123	9.88
LAV	Estimate	-0.0151	0.9928	
	Stand. Err	0.1262	0.0084	6.73
MM	Estimate	-0.009	0.9871	
	Stand. Err	0.0175	0.0012	0.935
S	Estimate	-0.0021	0.9867	
	Stand. Err	0.025	0.0074	1.62
M-BISQUARE	Estimate	-0.0083	0.9865	
	Stand. Err	0.0152	0.001	0.81
M-HUBBER	Estimate	-0.0199	0.9929	
	Stand. Err	0.0224	0.0015	1.195

The average precision error estimates that depends on the parameters (β_0, β_1) shows that the M-BISQUARE has the least precision error estimates of 0.81% with OLS having the highest precision error estimates of 9.88%. The LAV, S, M-HUBER and MM follows in that order with a precision errors of 6.73%, 1.62%, 1.195% and 0.935% respectively. Thus, it can be concluded that all the robust regression techniques discussed (S, M-HUBER, MM and M-BISQUARE) gave an acceptable precision error estimates apart from OLS and LAV. This anomaly in solution of the precision error estimates of OLS and LAV can be attributed to the uncontrollable effect of the outliers in the data.

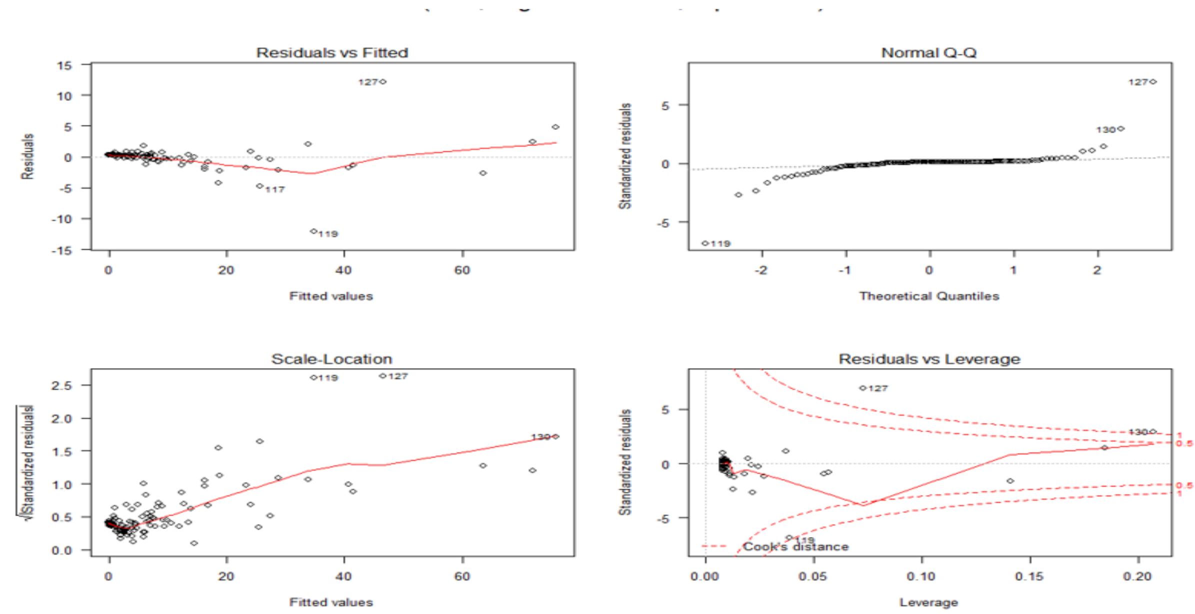

Figure 3. Diagnostic plot for OLS

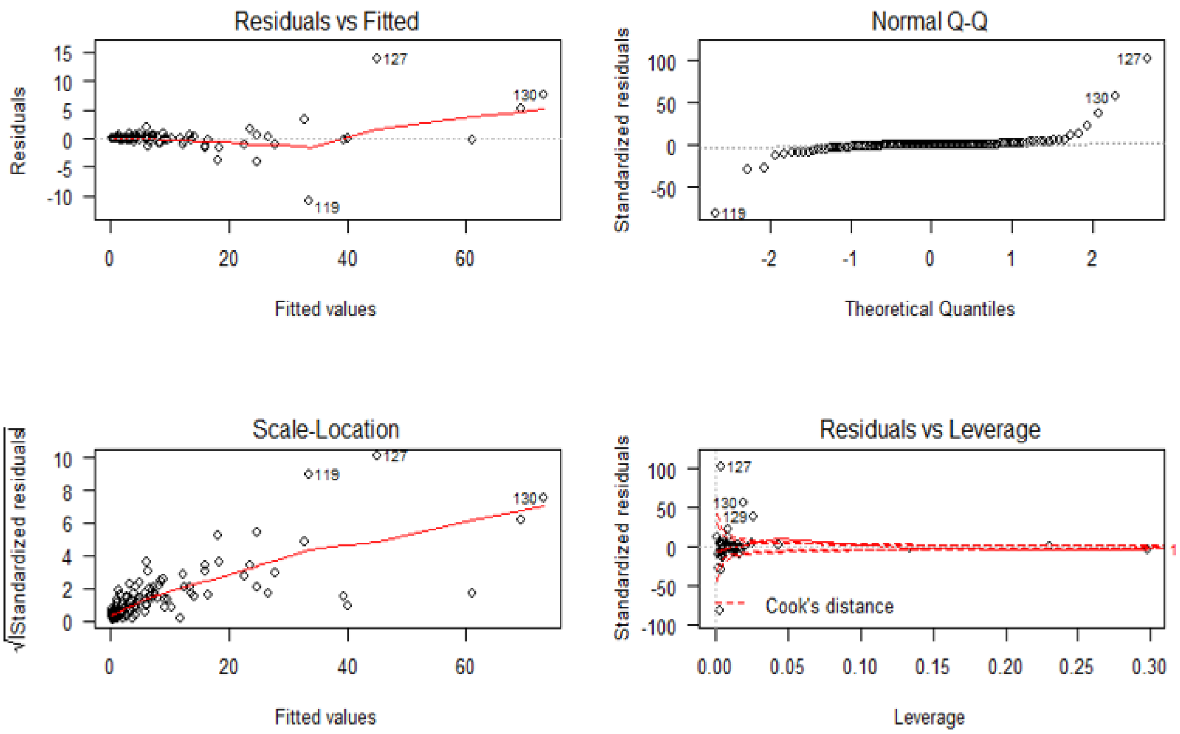

Figure 4. Diagnostic plot for M-BISQUARE

The optimal precision estimates for the robust regression techniques explored on the mine assay data was the M-BISQUARE with a percentage precision estimate value of about 99%. A diagnostic plot of the OLS and the M-BISQUARE methods to assess the residual effect on the fitted and the leverage are shown in Figures 3 and 4. The residual-fitted plot of the OLS and the M-BISQUARE in Figures 3 and 4 shows that the M-BISQUARE fitted plot is better conditioned with virtually no extreme outliers, but that of the OLS plot have many extreme outliers. The residual-leverage plot on the other hand shows that the M-BISQUARE is more streamlined toward the trendline as compared to the OLS with a number of uncorrelated contours, which are not directed towards the trendline

Thus, from the M-BISQUARE plot in Figure 4 above, it is evident that the M-BISQUARE reduces the effect of outliers in the data, which in effect reduces the error in solution yielding a much better parameter estimates than the rest of the methods applied**.**

4. Conclusions

The optimal precision estimates for the robust regression techniques explored on the mine assay data was the M-BISQUARE with a percentage precision estimate value of about 99%. A diagnostic plot of the OLS and the M-BISQUARE methods to assess the residual effect on the fitted and the leverage are shown in Figures 3 and 4. The residual-fitted plot of the OLS and the M-BISQUARE in Figures 3 and 4 shows that the M-BISQUARE fitted plot is better conditioned with virtually no extreme outliers, but that of the OLS plot have many extreme outliers. The residual-leverage plot on the other hand shows that the M-BISQUARE is more streamlined toward the trendline as compared to the OLS with a number of uncorrelated contours, which are not directed towards the trendline

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References

Abzalov M. *Sampling Errors and Control of Assay Data Quality in Exploration and Mining Geology*, IntechOpen, 2007: 307-236.

Abzalov, M. Geostatistical approach to the estimation of sampling precision. *The 5th World Conference on Sampling and Blending, Chile (materiały konferencyjne)* (2011).

Abzalov, M.Z. Gold deposits of the Russian North East (the Northern Circum Pacific): Metallogenic overview, *in* AusIMM, eds., *Proceedings of the PACRIM '99 symposium*: Australasian Institute of Mining and Metallurgy, pp. 701–714, 1999.

Abzalov, M.Z., and Both, R.A. The Pechenga Ni-Cu deposits, Russia: Data on PGE and Au distribution and sulphur isotope compositions, *Mineralogy and Petrology*, 61(1997): 119–143.

Abzalov, M.Z., and Humphreys, M. Resource estimation of structurally complex and discontinuous mineralisation using non-linear geostatistics: Case study of a mesothermal gold deposit in northern Canada, *EMG*, 11(2002): 19–29.

Abzalov, M.Z., and Mazzoni, P. The use of conditional simulation to assess process risk associated with grade variability at the Corridor Sands detrital ilmenite deposit, *in* Dimitrakopoulus, R., and Ramazan, S., eds., Ore body modelling and strategic mine planning: Uncertainty and risk management. *AusIMM*, (2004): 93–101.

Abzalov, M.Z., and Pickers, N. Integrating different generations of assays using multivariate geostatistics: A case study: Transactions of Institute of Mining and Metallurgy, Section B: *Applied AES*, 114(2005): B23–B32.

Abzalov, M.Z., Menzel, B., Wlasenko, M., and Phillips, J. Grade control at Yandi iron ore mine, Pilbara region, Western Australia—Comparative study of the blastholes and Reverse Circulation holes sampling, *AusIMM*, 2007: 37–43.

Cohen, J., Cohen, P., West, S.G. and Aiken, L.S. *Applied multiple regression/correlation analysis for the behavioral sciences*. Lawrence Erl- baum Associates. Mahwah, *NJ*, 2003.

Davis, J.C. *Statistics and data analysis in geology*, 3rd ed.: New York, John Wiley and Sons, pp.1-638, 2002.

Dominy, S., O'Connor, L., Glass, H., Purevgerel, S. and Xie, Y. Towards Representative Metallurgical Sampling and Gold Recovery Testwork Programmes. *Minerals*, 8(5) (2018): 1- 193.

Eremeev, A.N., Ostroumov, G.V., Anosov, V.V., Berenshtein, L.E., Korolev, V.P., and Samonov, I.Z. Instruction on internal, external, and arbitrary quality control of the exploration samples assayed in the laboratories of the Ministry of Geology of the USSR, *VIMS*, 1982: 106-106.

François-Bongarçon D. The practice of the sampling theory of broken ores. *ClM Bulletin*, 86(970)(1993): 75-81.

Goovaerts, P. *Geostatistics for natural resources evaluation*: New York, Oxford University Press, pp. 1- 483, 1997.

Gy P.M. *Sampling of particulate materials, theory and practice.* 2nd edition, Amsterdam: Elsevier, 1982

Huber Jr, E.E. and Ridgley, D.H. Magnetic properties of a single crystal of manganese phosphide. *PR*, 135(4A) (1964): A1-A1033.

ISO Guide 33, 1989, *Uses of certified reference materials*: Standards Council of Canada, 12 p.

Koenker, R. and Bassett Jr, G., Regression quantiles. *Econometrica: JES*, (1978): 33-50.

Leaver, M.E., Sketchley, D.A., and Bowman, W.S. The benefits of the use of CCRMP's custom reference materials: Canadian certified reference materials project: Society of Mineral Analysts Conference, *MSL*, 637(16) (1997).

Long, S. Practical quality control procedures in mineral inventory estimation: *EMG*, 7(1998): 117–127.

McAlinden, C., Khadka, J. and Pesudovs, K. Precision (repeatability and reproducibility) studies and sample-size calculation. *JCRS*, 41(12) (2015): 2598-2604.

Pitard F. F., *Pierre Gy's Sampling Theory and Sampling Practise: Heterogeneity, Sampling Correctness, and Statistical Process Control*, 2nd ed.; CRC Press, New York, pp.1- 488, 1993.

Rousseeuw, P. and Yohai, V. Robust regression by means of S-estimators. In *Robust and nonlinear time series analysis*. Springer, New York, pp. 256-272, 1984.

Samkar, H. and Alpu, O. Ridge regression based on some robust estimators. *JMASM*, 9(2) (2010): 1-17.

Sinclair, A.J., and Bentzen, A. Evaluation of errors in paired analytical data by a linear model: *EMG*, 7(1-2) (1998): 167–173.

Sinclair, A.J., and Blackwell, G.H. *Applied mineral inventory estimation*, Cambridge University Press, pp. 1-381, 2002.

Sketchley, D.A. Gold deposits: Establishing sampling protocols and monitoring quality control, *EMG*, 7(1-2) (1988): 129–138.

Stanley, C.R., and Lawie, D. Average relative error in geochemical determinations: Clarification, calculation, and a plea for consistency, *EMG*, 16(2007a); 265–274.

Susanti, Y. and Pratiwi, H. M estimation, S estimation, and MM estimation in robust regression. *IJPAM*, *91*(3) (2014): 349-360.

Taylor, J.K. Quality assurance of chemical measurements: Michigan, Lewis Publishers, pp. 1-135, 1987.

Vallée, M., David, M., Dagbert, M., and Desrochers, C. Guide to the evaluation of gold deposits, *Special* 45(1992): 299-299.

Yohai, V.J. High breakdown-point and high efficiency robust estimates for regression. *The Ann. Stat.*, *15*(2) (1987): 642-656.

APPENDIX: ASSAY DATA COLLECTED FROM EXPLORATION MINE

No.	Orig.	Dup.		Orig.	Dup.		Orig.	Dup.	No.	Orig.	Dup.	No.	Orig.	Dup
1	0.22	0.2	27	0.5	0.53	53	1.5	1.64	79	4.57	4.89	105	10.3	10.5
2	0.23	0.22	28	0.55	0.55	54	1.6	1.75	80	4.62	4.09	106	11	12.2
3	0.23	0.21	29	0.55	0.53	55	1.7	1.26	81	4.74	4.61	107	11.7	11.8
4	0.24	0.24	30	0.55	0.6	56	1.9	1.98	82	4.89	4.72	108	11.9	12.6
5	0.24	0.27	31	0.56	0.53	57	1.9	2.13	83	5.07	6.38	109	13.3	13.8
6	0.24	0.24	32	0.57	0.54	58	2.0	2.12	84	5.76	5.02	110	13.9	13.4
7	0.24	0.27	33	0.58	0.62	59	2.0	1.95	85	5.79	6	111	14.3	16
8	0.26	0.24	34	0.59	0.58	60	2.1	2.26	86	5.88	5.9	112	14.4	14.2
9	0.26	0.25	35	0.59	0.59	61	2.2	2.31	87	5.95	6.56	113	14.4	18.3
10	0.26	0.27	36	0.62	0.65	62	2.3	2.32	88	6.15	6.1	114	14.6	16
11	0.26	0.29	37	0.66	0.68	63	2.4	2.78	89	6.15	5.8	115	16	16.5
12	0.28	0.23	38	0.85	0.83	64	2.4	2.82	90	6.5	6.9	116	16.5	18.4
13	0.29	0.34	39	0.96	0.99	65	2.6	2.66	91	6.76	6.5	117	20.8	25
14	0.3	0.3	40	0.99	1	66	2.6	2.74	92	7.05	7.5	118	21.5	22.7
15	0.3	0.32	41	1	1.03	67	2.6	2.66	93	7.2	7.62	119	22.6	33.8
16	0.3	0.33	42	1.04	1.04	68	3.0	3.31	94	7.29	7.1	120	25	23.6
17	0.3	0.34	43	1.05	1.05	69	3.1	3.12	95	7.5	7.1	121	25.2	24.8
18	0.34	0.37	44	1.06	0.98	70	3.2	3.26	96	7.55	8.3	122	26.7	28.1
19	0.34	0.39	45	1.09	1.08	71	3.5	3.61	97	7.62	8.5	123	26.9	26.7
20	0.36	0.34	46	1.09	1.08	72	3.5	4.05	98	7.74	6	124	35.9	33
21	0.36	0.4	47	1.23	1.6	73	3.8	3.16	99	7.75	7.2	125	39	39.6
22	0.38	0.36	48	1.29	1.07	74	3.9	3.74	100	8.11	7.9	126	40.2	40.4
23	0.4	0.4	49	1.3	1.41	75	4.0	4.12	101	8.84	9.18	127	58.8	45.3
24	0.46	0.44	50	1.32	1.18	76	4.1	4.27	102	9.11	9.3	128	60.8	61.6
25	0.46	0.48	51	1.34	1.45	77	4.1	4.1	103	9.41	9.73	129	74.4	69.8
26	0.47	0.51	52	1.44	1.48	78	4.4	4.46	104	9.7	8.9	130	80.7	73.6