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Article

On Some Characterizations of Generalized Log Pearson Type-VII Distribution

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Abstract: In this article, generalized log Pearson type VII (GLPT-VII) distribution is characterized via (i) doubly truncated moments and (ii) ratio of truncated moments. The applications and utility of characterizations of GLPT-VII distribution will be constructive for researchers in different disciplines of science.

Keywords: Characterization; Truncated; Moments

Mathematics Subject Classification: 62E10, 62E15

1. Introduction

Pearson [20, 21, 22] presented a system of probability distributions that have wide applications in reliability, manufacturing, genetic and medical sciences, mathematical economics, life testing and survey sampling. Pearson models are popular in monetary markets and capture the stochastic nature of the unpredictability of rates and stocks.

Habibullah [9] presented log Pearson type VII (LPT-VII) distribution along with its properties. Iqbal and Ahmad [17] developed GLPT-VII distribution along with its some properties. Lahcene [18] worked on Pearson families of distributions.

1.1. Generalized Log Pearson Type VII (GLPT-VII) Distribution

If random variable X follows GLPT-VII(α,β,γ) distribution, then probability density function (pdf) X is

$$f(x;\alpha,\beta,\gamma)=\frac{\beta\alpha^{\frac{1}{2\beta}}}{B\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)}\frac{1}{x}\left(1+\alpha(\ln x)^{2\beta}\right)^{-\gamma} \quad ,x>0, \tag{1}$$

where $\gamma>\frac{1}{2\beta}$, $x>0,\alpha>0,\beta\in Z^{+}$ and $B\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)$ is a complete beta function. Now α is scale parameter, β and γ, are the shape parameters. For $\beta=1$, $GLPT-VII(\alpha,\beta,\gamma)$ distribution becomes two parameters $LPT-VII(\alpha,\gamma)$ distribution.

The cumulative distribution function (cdf) of X is

$$F(x;\alpha,\beta,\gamma)=\begin{cases}\frac{1}{2}B_{\left(1+\alpha(\ln x)^{2\beta}\right)^{-1}}\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right), & if\ \ 0<x\le 1\\ 1-\frac{1}{2}B_{\left(1+\alpha(\ln x)^{2\beta}\right)^{-1}}\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right), & if\ \ x>1\end{cases} \tag{2}$$

where $B_{\left(1+\alpha(\ln x)^{2\beta}\right)^{-1}}\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)$ is incomplete beta function.

2. Characterizations

In order to develop a stochastic function for a certain problem, it is necessary to know whether function fulfills the theory of specific underlying probability distribution, it is required to study characterizations of specific probability distribution. Different characterization techniques have developed. Glänzel ([5,6,7]), Ahsanullah and Hamedani [1], Glänzel and Hamedani [8] Hamedani ([10-15]), Hamedani and Ahsanullah [16], Merovci et al. [19], Bhatti et al. [3], Bhatti et al. [4] and Bhatti and Ali [2] have worked on various techniques of characterizations.

In this article, GLPT-VII distribution is characterized via (i) doubly truncated moments and (ii) ratio of truncated moments.

2.1. Characterization through Doubly Truncated Moments

Here GLPT-VII distribution is characterized via doubly truncated moments.

Proportion 2.1.1: Let $X:\Omega \to (0,\infty)$ be a GLPT-VII random variable. Then X has pdf (1), if and only if

$$E\left[(\ln x)^{2\beta-1}\middle| x<X<y\right]=\frac{\left(1+\alpha(\text{lny})^{2\beta}\right)yf(y)-\left(1+\alpha(\text{lnx})^{2\beta}\right)xf(x)}{2\beta\gamma(1-\gamma)\left[F(y)-F(x)\right]}. \quad (3)$$

Proof

For a GLPT-VII random variable X with pdf (1), we have

$$E\left[(\ln x)^{2\beta-1}\middle| x<X<y\right]=\frac{\int_y^x(\text{lnu})^{2\beta-1}f(u)du}{\left[F(y)-F(x)\right]},$$

$$=\frac{\int_y^x 2\beta\alpha(\text{lnu})^{2\beta-1}\frac{k}{u}\left(1+\alpha(\text{lnu})^{2\beta}\right)^{-\gamma}du}{2\beta\alpha\left[F(y)-F(x)\right]},$$

$$=\frac{\left(1+\alpha(\text{lnu})^{2\beta}\right)uf(u)\Big|_x^y}{(1-\gamma)2\beta\alpha\left[F(y)-F(x)\right]},$$

$$E\left[(\ln x)^{2\beta-1}\middle| x<X<y\right]=\frac{\left(1+\alpha(\text{lny})^{2\beta}\right)yf(y)-\left(1+\alpha(\text{lnx})^{2\beta}\right)xf(x)}{2\beta\alpha(1-\gamma)\left[F(y)-F(x)\right]}.$$

Conversely, if (3) holds, then

$$\frac{\int_y^x(\text{lnu})^{2\beta-1}f(u)du}{\left[F(y)-F(x)\right]}=\frac{\left(1+\alpha(\text{lnu})^{2\beta}\right)uf(u)\Big|_x^y}{(1-\gamma)2\beta\alpha\left[F(y)-F(x)\right]},$$

$$2\beta\alpha(1-\gamma)E\int_x^y(\text{lnu})^{2\beta-1}f(u)du=\left(1+\alpha(\text{lny})^{2\beta}\right)yf(y)-\left(1+\alpha(\text{lnx})^{2\beta}\right)xf(x). \quad (4)$$

Differentiate (4) w.r.t y, we obtained

$$2\beta\alpha(1-\gamma)(\text{lny})^{2\beta-1}f(y)=2\beta\alpha(\text{lny})^{2\beta-1}f(y)+\left(1+\alpha(\text{lny})^{2\beta}\right)f(y)+\left(1+\alpha(\text{lny})^{2\beta}\right)y\,f'(y),$$

or

$$yf'(y)\left[1+\alpha(\ln y)^{2\beta}\right]=f(y)\left[-\left(1+\alpha(\ln y)^{2\beta}\right)-2\beta\alpha\gamma(\ln y)^{2\beta-1}\right],$$

or

$$\frac{d}{dx}\left[\ln f(y)\right]=\left[-\frac{1}{y}-\frac{2\beta\alpha\gamma(\ln y)^{2\beta-1}}{y\left(1+\alpha(\ln y)^{2\beta}\right)}\right]. \qquad (5)$$

Integrate (5), we obtained

$$f(y)=ke^{\ln\left[\frac{1}{y}\left(1+\alpha(\ln y)^{2\beta}\right)^{-\gamma}\right]},$$

$$f(y;\alpha,\beta,\gamma)=\frac{\beta\alpha^{\frac{1}{2\beta}}}{B\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)}\frac{1}{y}\left(1+\alpha(\ln y)^{2\beta}\right)^{-\gamma},\ y>0,$$

is the pdf of GLPT-VII distribution.

2.2. Characterization of GLPT-VII via Ratio of Truncated Moments

GLPT-VII distribution is characterized from simple relationship between two truncated moments of functions of X. [Theorem G (Glänzel; [5]].

Proportion 2.2.1: Let $X:\Omega\to(0,\infty)$ be GLPT-VII random variable. Let

$$h_1(x)=2\alpha^{1-\frac{1}{2\beta}}mB\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)(\ln x)^{2\beta-1}\left(1+\alpha(\ln x)^{2\beta}\right)^{\gamma-m-1},x>0,$$

and

$$h_2(x)=2\alpha^{1-\frac{1}{2\beta}}(m+1)B\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)(\ln x)^{2\beta-1}\left(1+\alpha(\ln x)^{2\beta}\right)^{\gamma-m-2},x>0,\ \gamma>\frac{1}{2\beta}.$$

The random variable X has pdf (1), if and only if the function $p(x)$ (defined in Theorem G) has the simple form $p(x)=1+\alpha(\ln x)^{2\beta}$.

Proof

For a GLPT-VII random variable X with pdf (1), then

$$\left(1-F(x)\right)E\left(h_1(x)\middle| X\ge x\right)=\left(1+\alpha(\ln x)^{2\beta}\right)^{-m},\ x>0,$$

$$\left(1-F\left(x\right)\right)E\left(h_2\left(x\right)\middle| X\geq x\right)=\left[1+\alpha\left(\ln x\right)^{2\beta}\right]^{-(m+1)},x>0,$$

$$\frac{E[h_1\left(x\right)| X\geq x]}{E[h_2\left(x\right)| X\geq x]}=p\left(x\right)=1+\alpha\left(\ln x\right)^{2\beta},x>0,$$

and

$$p'\left(x\right)=\frac{2\alpha\beta\left(\ln x\right)^{2\beta-1}}{x},\quad x>0.$$

Conversely if $p\left(x\right)=1+\alpha\left(\ln x\right)^{2\beta},x>0$, then the differential equation

$$s'\left(x\right)=\frac{p'\left(x\right)h_2\left(x\right)}{p\left(x\right)h_2\left(x\right)-h_1\left(x\right)}=\frac{2\alpha\beta\left(m+1\right)\left(\ln x\right)^{2\beta-1}}{x\left(1+\alpha\left(\ln x\right)^{2\beta}\right)},x>0,$$

has solution

$$s\left(x\right)=\ln\left[1+\alpha\left(\ln x\right)^{2\beta}\right]^{m+1},x>0.$$

Therefore, in the view of theorem G, X has pdf (1).

Corollary 2.2.1: Let $X:\Omega\rightarrow\left(0,\infty\right)$ be GLPT-VII random variable and let

$$h_2\left(x\right)=2\alpha^{1-\frac{1}{2\beta}}\left(m+1\right)B\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)\left(\ln x\right)^{2\beta-1}\left(1+\alpha\left(\ln x\right)^{2\beta}\right)^{\gamma-m-2},x>0,\gamma>\frac{1}{2\beta}.$$

The pdf of X is (1), if and only if functions $p\left(x\right)$ and $h_1\left(x\right)$ (defined in Theorem G) satisfying equation

$$\frac{p'\left(x\right)}{p\left(x\right)h_2\left(x\right)-h_1\left(x\right)}=\frac{\beta\alpha^{\frac{1}{2\beta}}}{B\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)}\frac{1}{x}\left(1+\alpha\left(\ln x\right)^{2\beta}\right)^{-\gamma+m+1},\gamma>\frac{1}{2\beta}.\qquad(6)$$

Remark 2.2.1: The solution of (6) is

$$p\left(x\right)=\left(1+\alpha\left(\ln x\right)^{2\beta}\right)^{-(m+1)}\int\left[-h_1\left(x\right)\frac{\beta\alpha^{\frac{1}{2\beta}}}{B\left(\gamma-\frac{1}{2\beta},\frac{1}{2\beta}\right)}\frac{1}{x}\left(1+\alpha\left(\ln x\right)^{2\beta}\right)^{-\gamma+2m+2}\right]dx+K,$$

where K is constant.

3. Conclusion

We have characterized GLPT-VII distribution via: (i) doubly truncated moments and (ii) ratio of truncated moments.

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